Limit Laws Assume that $\lim_{(x,y)\to P} f(x,y)$ and $\lim_{(x,y)\to P} g(x,y)$ exist. Then:

(i) Sum Laws:

$$\lim_{(x,y)\to P} (f(x,y) + g(x,y)) = \lim_{(x,y)\to P} f(x,y) + \lim_{(x,y)\to P} g(x,y)$$

(ii) Constant Multiple Law:

$$\lim_{(x,y)\to P} kf(x,y) = k \lim_{(x,y)\to P} f(x,y)$$

(iii) Product Law:

$$\lim_{(x,y)\to P} f(x,y)g(x,y) = \left(\lim_{(x,y)\to P} f(x,y)\right)\left(\lim_{(x,y)\to P} g(x,y)\right)$$

(iv) Quotient Law:

If $\lim_{(x,y)\to P} g(x,y) \neq 0$, then

$$\lim_{(x,y)\to P} \frac{f(x,y)}{g(x,y)} = \frac{\lim_{(x,y)\to P} f(x,y)}{\lim_{(x,y)\to P} g(x,y)}$$

Definition of Continuity

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$$

1. Show that $f(x,y) = \frac{3x+y}{x^2+y^2+1}$ is continuous and evaluate $\lim_{(x,y)\to(1,2)} f(x,y)$

only chance for discontinuity $\to$ denom $= 0$

x^2+y^2+1 always positive so $\neq 0$

$x^2+y^2 \neq -1$

$$\lim_{(x,y)\to(1,2)} f(x,y) = \frac{3(1)+2}{1^2+2^2+1} = \frac{5}{6}$$

2. Evaluate $\lim_{(x,y)\to(3,0)} x^3 \frac{\sin y}{y}$

$$\left(\lim_{(x,y)\to(3,0)} x^3\right)\left(\lim_{(x,y)\to(3,0)} \frac{\sin y}{y}\right)$$

remember from Calc $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$= (27)(1)$$

$$= 27$$

A Composition of Continuous Functions is Continuous
If a function of two variables f is continuous at (a, b) and a function of one variable G is continuous at $c = f(a, b)$, then the composite function $G(f(x, y))$ is continuous at (a, b)

3. Write $H(x, y) = e^{-x^2+2y}$ as a composite function and evaluate $\lim_{(x,y)\to(1,2)} H(x, y)$

$f(x) = e^x$ $\quad g(x,y) = -x^2 + 2y$ $\quad H(x,y) = f(g(x,y))$

$$\lim_{(x,y)\to(1,2)} e^{-x^2+2y} = e^{-1+2(2)}$$

$$= e^3$$

4. $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$ = DNE

concerned $x^2 = -y^2$ $\quad$ try to evaluate using substition get $\frac{0}{0}$

attempt to reason w/ approaching values
↳ make a table of values

$\lim_{(x,y)\to(0.1,0)} \frac{x^2}{x^2+y^2} = 1$ $\quad \lim_{(x,y)\to(0,0.1)} \frac{x^2}{x^2+y^2} = 0$ $\quad \lim_{(x,y)\to(0.1,0.1)} \frac{x^2}{x^2+y^2} = 0.5$

does not apprear to approach same value