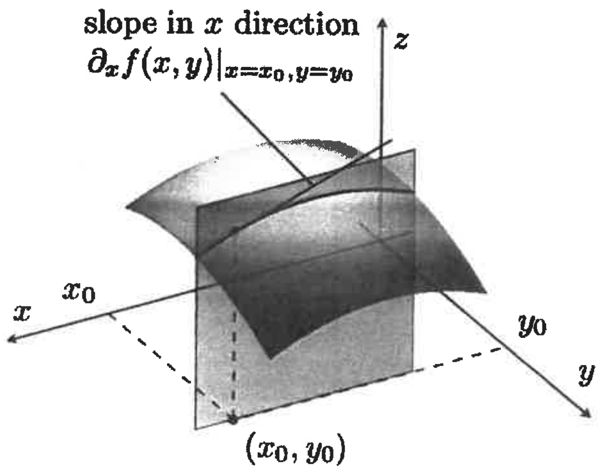
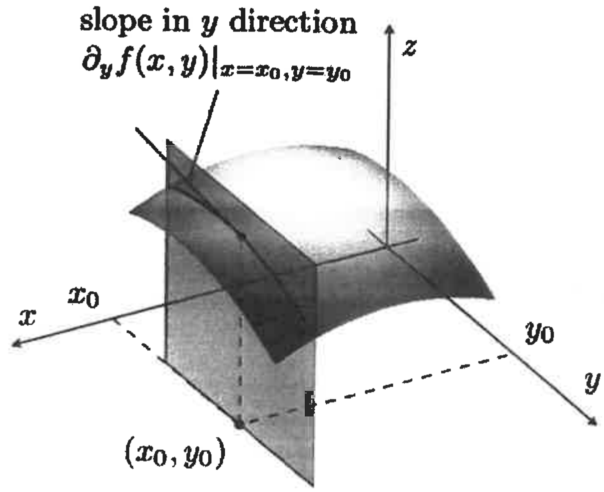
Partial derivatives are computed just like ordinary derivatives in one variable with this difference: To compute f_x, treat y as a constant, and to compute f_y, treat x as a constant.





1. Compute the partial derivatives of $f(x,y) = x^2y^5$

$f_x(x,y) = 2xy^5$

$f_y(x,y) = 5x^2y^4$

2. Calculate $g_x(1,3)$ and $g_y(1,3)$, where $g(x,y) = \frac{y^2}{(1+x^2)^3}$

$g_x(x,y) = y^2\left(-3(1+x^2)^{-4}(2x)\right)$

$= \frac{-6xy^2}{(1+x^2)^4}$

$g_x(1,3) = \frac{-6(1)(3)^2}{(1+1^2)^4}$

$= \frac{-54}{16}$

$= \boxed{-\frac{27}{8}}$

$g_y(x,y) = \frac{1}{(1+x^2)^3}\, 2y$

$= \frac{2y}{(1+x^2)^3}$

$g_y(1,3) = \frac{2(3)}{(1+1^2)^3}$

$= \frac{6}{8}$

$= \boxed{\frac{3}{4}}$

3. Calculate $\frac{d}{dx}\sin(x^2y^5)$

$2xy^5\cos(x^2y^5)$

4. Calculate $f_z(0,0,1,1)$, where $f(x,y,z,w) = \frac{e^{xz+y}}{z^2+w}$

$$f_z(x,y,z,w) = \frac{(z^2+w)\,e^{xz+y}\,x - e^{xz+y}\,2z}{(z^2+w)^2}$$

$$f_z(0,0,1,1) = \frac{(1^2+1)\,e^{0(1)+0}\,(0) - e^{0(1)+0}\,2(1)}{(1^2+1)^2} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$$

5. Calculate the second-order partial derivatives of $f(x,y) = x^3 + y^2e^x$

$f_x(x,y) = 3x^2 + y^2e^x$	$f_y(x,y) = 2ye^x$
$f_{xx}(x,y) = 6x + y^2e^x$	$f_{yy}(x,y) = 2e^x$
$f_{xy}(x,y) = 2ye^x$	$f_{yx}(x,y) = 2ye^x$

6. Calculate f_{xyy} for $f(x,y) = x^3 + y^2e^x$

$$f_x(x,y) = 3x^2 + y^2e^x$$

$$f_{xy}(x,y) = 2ye^x$$

$$f_{xyy}(x,y) = 2e^x$$

Clairut's Theorem: Equality of Mixed Partials
If f_{xy} and f_{yx} are both continuous functions on a disk D, then $f_{xy}(a,b) = f_{yx}(b,a)$ for all $(a,b) \in D$.

$$\frac{d^2f}{dxdy} = \frac{d^2f}{dydx}$$

7. Calculate the partial derivative g_{zzwx} where $g(x,y,z,w) = x^3w^2z^2 + \sin(\frac{xy}{z^2})$

★ choose order wisely ★

$$g_w(x,y,z,w) = x^3 z^2 (2w)$$
$$= 2x^3z^2w$$

$$g_{wz}(x,y,z,w) = 4x^3zw$$

$$g_{wzz}(x,y,z,w) = 4x^3w$$

$$g_{wzzx}(x,y,z,w) = 12x^2w$$

$$g_{zzwx}(x,y,z,w) = 12x^2w$$