*locally linear - if its graph looks flatter and flatter as we zoom in on point $P(a, b, f(a, b))$

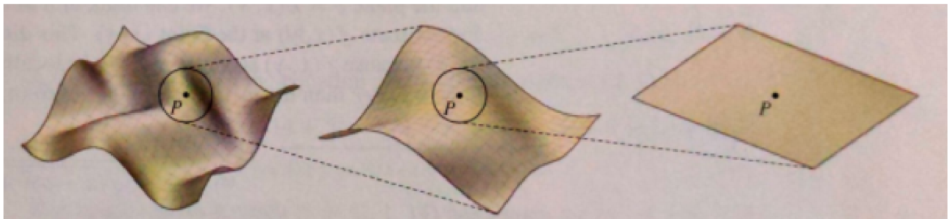

FIGURE 2 The graph looks flatter and flatter as we zoom in on a point P.

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Equation of the Tangent Plane

If $f(x, y)$ is locally linear at (a, b), then its tangent plane is given by the equation

$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

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FIGURE 1 Tangent plane to the graph of $z = f(x, y)$.

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1. Given that $f(x, y) = 5x + 4y^2$ is differentiable. Find the equation of the tangent plane at $(2, 1)$.

2. Find a tangent plane of the graph of $f(x, y) = xy^3 + x^2$ at $(2, -2)$.

3. What is the equation of the tangent plane at (1,1) to the surface $4 - x^2 - y^2 = z$.

    a. Estimate $f(1.1, 0.9)$ given $f(x, y) = z$

Recall how we approximated function values in 2d:

This idea continues into multiple variables and dimensions so long as it is locally linear. Try to write an equation that represents a linear approximation in 3d.

We can also write the linear approximation in term of the change in f: $\Delta f = f(x, y) - f(a, b)$

$$\Delta f \approx f_x(a, b)\Delta x + f_y(a, b)\Delta y$$

$$\Delta f \approx df, \quad \Delta x \approx dx, \quad \Delta y \approx dy$$

$$df = f_x(x, y)\Delta x + f_y(x, y)\Delta y = \frac{df}{dx}dx + \frac{df}{dy}dy$$

4. Use the linear approximation to estimate $(3.93)^3(1.01)^4(1.98)^{-1}$

5. Use differentials to find an approximate value for $\sqrt{1.03^2 + 1.98^3}$

6. Find the total differential of $W = x^5y^3 + x^2z^4$

7. Estimate the amount of material in a closed can (right circular cylinder) with a radius of 3 inches and a height of 8 inches if the material of the can is 0.04 inches thick.