The rate of change of a function f of several variables depends on a choice of direction. Since directions are indicated by vectors, it is natural to use vectors to describe the derivative of f in a specified direction.

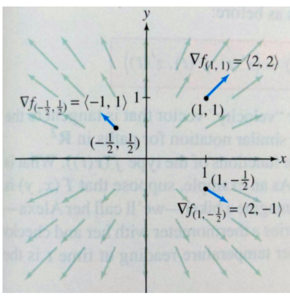
The Gradient The gradient of a function $f(x, y)$ at a point $P = (a, b)$ is the vector

$$\nabla f_p = \langle f_x(a,b), f_y(a,b) \rangle$$

In three variables, if $P = (a, b, c)$

$$\nabla f_p = \langle f_x(a,b,c), f_y(a,b,c), f_z(a,b,c) \rangle$$

1. Let $f(x, y) = x^2 + y^2$. Calculate the gradient ∇f. Compute ∇f_P at $P = (1, 1)$



2. Calculate $\nabla f_{(3,-2,4)}$, where $f(x, y, z) = ze^{2x+3y}$

Properties of the Gradient If $f(z, y, z)$ and $g(x, y, z)$ are differentiable and c is a constant, then:

1. $\nabla(f+g) = \nabla f + \nabla g$
2. $\nabla(cf) = c\nabla f$
3. Product Rule for Gradients: $\nabla(fg) = f\nabla g + g\nabla f$
4. Chain Rule for Gradients: If $F(t)$ is a differentiable function of one variable, then $\nabla(F(f(x,y,z))) = F'(F(x,y,z))\nabla f$

3. Find the gradient of $g(x, y, z) = (x^2 + y^2 + z^2)^8$

Chain Rule with Partial Derivatives

4. $z = 3x - 7y,\ x = \cos t,\ y = \sin t$, find $\frac{dz}{dt}$

5. $z = x^2 - 3xy,\ x = 5t^2 + 4,\ y = 3t$, find $\frac{dz}{dt}$

6. $f(x, y) = xe^y,\ \mathbf{r}(t) = \langle t^2, t^2 - 4t\rangle$, find $\frac{d}{dt} f(\mathbf{r}(t))$

7. $f(x, y) = \ln x + \ln y,\ \mathbf{r}(t) = \langle \cos(t), t^2 \rangle$, find $\frac{d}{dt} f(\mathbf{r}(\frac{\pi}{4}))$

8. $g(x, y, z) = xyz^{-1},\ x = e^t,\ y = t,\ z = t^2$, find $\frac{d}{dt} g(x, y, z)$ when $t = 1$

9. The temperature at location (x, y) is $T(x, y) = 20 + 10e^{-0.3(x^2+y^2)}$ $^\circ C$. A bug carries a tiny thermometer along the path $\mathbf{r}(t) = \langle \cos(t-2), \sin 2t \rangle$ (t in seconds). How fast is the temperature changing at $t = 0.6$ seconds?