Absolute Maximum and Absolute Minimum

Definition: A function f has a **absolute maximum** at (a, b) if $f(a, b)$ is the largest function value for the domain of f. Similarly, f has a **absolute minimum** at (a, b) if $f(a, b)$ is the smallest function value for the domain of f.

Extreme Value Theorem for Functions of One Variable: If f is continous on a closed interval $[a, b]$, then f has an absolute maximum and an absolute minimum value. These are found by evaluating critical points and the endpoints of the interval.

Extreme Value Theorem for Functions of Two Variables: If f is continuous on a closed and bounded set D in $\Re^2$, then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points (x_1, y_1) and (x_2, y_2) in D.

Definition: A **closed set** in $\Re^2$ is one that contains all of its boundry points.

Definition: A **bounded set** in $\Re^2$ is one that is contained in some disk.

To find the absolute maximum and minimum values of a continuous function f on a closed, bounded set D:

(1) Find the values of f at the critical points in D.
(2) Find the extreme values of f on the boundry of D.
(3) The largest of the values is the absolute maximum value; the smallest is the absolute minimum value.

Example: Find the absolute maximum/absolute minimum of f on the set D.

$D = \{(x, y) |\ 0 \leq x \leq 3, \quad -2 \leq y \leq 4 - 2x\}$

$f(x, y) = 4(x^2 + xy + 2y^2 - 3x + 2y) + 10$

Example: Find the absolute max for $f(x, y) = xy$ on the set D.

$$D = \left\{ (x, y) \,\middle|\, \frac{x^2}{16} + y^2 \leq 1 \right\}$$