Fubini's Theorem for Triple Integrals: The triple integral of a continuous function $f(x, y, z)$ over a box $B = [a, b] \times [c, d] \times [p, q]$ is equal to the iterated integral:

$$\iiint_B f(x, y, z)dV = \int_{x=a}^{b} \int_{y=c}^{d} \int_{z=p}^{q} f(x, y, z)dzdydx$$

Furthermore, the iterated integral may be evaluated in any order.

1. Evaluate $\iiint_B x^2 e^{y+3z} dV$, where $B = [1, 4] \times [0, 3] \times [2, 6]$

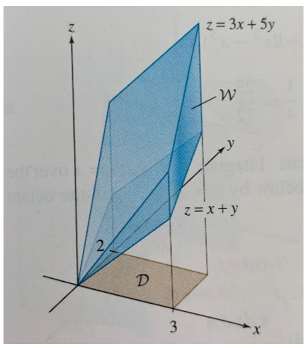
The triple integral of a continuous function f over the region

$$W:\ (x, y) \in D,\ \ z_1(x, y) \le z \le z_2(x, y)$$

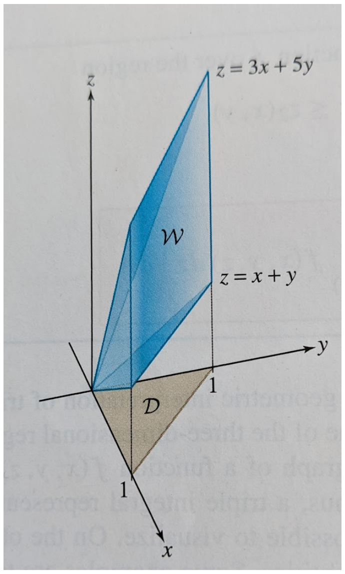
Is equal to the iterated integral

$$\iiint_W f(x, y, z)\,dV = \iint_D \left(\int_{z=z_1(x,y)}^{z=z_2(x,y)} f(x, y, z)\,dz \right) dA$$

2. Evaluate $\iiint_W z\,dV$, where W is the region between the planes $z = x + y$ and $z = 3x + 5y$ lying over the rectangle $D = [0, 3] \times [0, 2]$



3. Evaluate $\iiint\limits_{W} z\, dV$, where W is the region between the planes $z = x + y$ and $z = 3x + 5y$ lying over the triangle shown in the figure below.



4. Integrate $f(x, y, z) = x$ over the region W bounded above by $z = 4 - x^2 - y^2$ and below by $z = x^2 + 3y^2$ in the octant $x \geq 0,\ y \geq 0,\ z \geq 0$

Step 1: Find the boundary of D