

16.4 **Integration in Polar**, Cylindrical, and Spherical Coordinates
Multivariable Calculus

Remember:

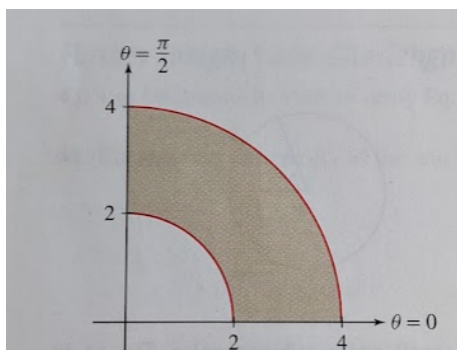
$$x = r \cos \theta \quad y = r \sin \theta$$

Double Integral in Polar Coordinates For a continuous function f on the domain

$$D: \theta_1 \leq \theta \leq \theta_2, \quad r_1(\theta) \leq r \leq r_2(\theta)$$

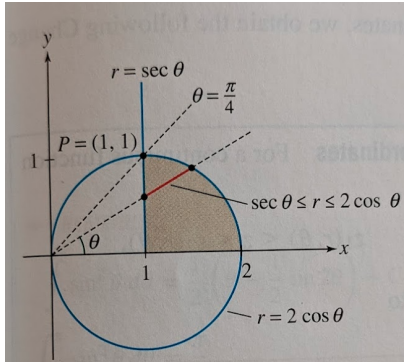
$$\iint_D f(x, y) dA = \int_{\theta_1}^{\theta_2} \int_{r=r_1(\theta)}^{r=r_2(\theta)} f(r \cos \theta, r \sin \theta) r \, dr \, d\theta$$

1. Compute $\iint_D (x + y) dA$, where D is the quarter annulus in the figure below:



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2. Calculate $\iint_D (x^2 + y^2)^{-2} dA$ for the shaded domain D in the figure below:



3. Evaluate $\iint_D 2xy dA$, D is the portion of the region between the circles of radius 2 and radius 5 centered at the origin that lies in the first quadrant.

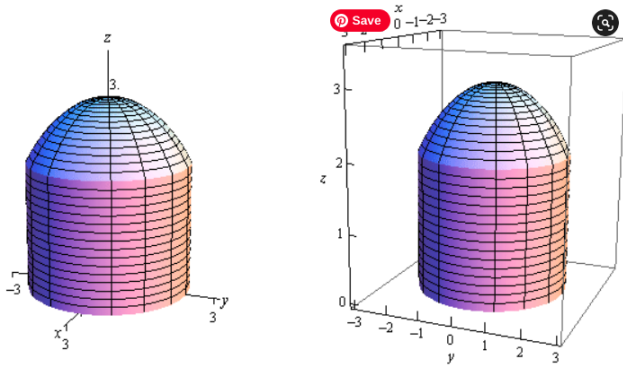
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4. Evaluate $\iint_D e^{x^2+y^2} dA$, D is the unit disk centered at the origin.

5. Determine the area of the region that lies inside $r = 3 + 2 \sin \theta$ and outside $r = 2$

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6. Determine the volume of the region that lies under the sphere $x^2 + y^2 + z^2 = 9$, above the plane $z = 0$ and inside the cylinder $x^2 + y^2 = 5$



7. Find the volume of the region that lies inside $z = x^2 + y^2$ and below the plane $z = 16$

