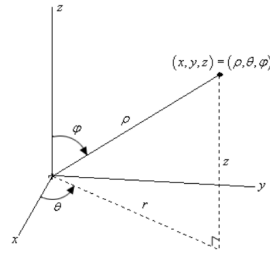


## 16.4 Integration in Spherical Coordinates Multivariable Calculus



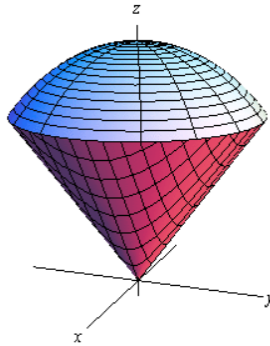
Conversion formulas for spherical coordinates:

$$x = \rho \sin \varphi \cos \theta \quad y = \rho \sin \varphi \sin \theta \quad z = \rho \cos \varphi$$

$$x^2 + y^2 + z^2 = \rho^2$$

$$\rho \geq 0 \quad 0 \leq \varphi \leq \pi$$

We will be slicing our solid into spherical wedges to calculate the integrals:



### Triple Integrals in Spherical Coordinates:

$$\iiint_E f(x, y, z) dV = \int_{\delta}^{\gamma} \int_{\alpha}^{\beta} \int_a^b \rho^2 \sin \varphi f(\rho \sin \varphi \cos \theta, \rho \sin \varphi \sin \theta, \rho \cos \varphi) d\rho d\theta d\varphi$$

Evaluate  $\iiint_E 16z dV$  where  $E$  is the upper half of the sphere  $x^2 + y^2 + z^2 = 1$ .

16.4 Integration in Spherical Coordinates  
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Evaluate  $\iiint_E 10xz + 3 \, dV$  where  $E$  is the region portion of  $x^2 + y^2 + z^2 = 16$  with  $z \geq 0$ .

Evaluate  $\iiint_E 3z \, dV$  where  $E$  is the region inside both  $x^2 + y^2 + z^2 = 1$  and  $z = \sqrt{x^2 + y^2}$ .

Evaluate  $\iiint_E z x \, dV$  where  $E$  is inside both  $x^2 + y^2 + z^2 = 4$  and the cone (pointing upward) that makes an angle of  $\frac{\pi}{3}$  with the negative  $z$ -axis and has  $x \leq 0$ .

16.4 Integration in Spherical Coordinates  
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Convert  $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} x^2 + y^2 + z^2 dz dx dy$  into spherical coordinates.