Video: https://www.youtube.com/watch?v=wUF-lyyWpUc

Suppose that we want to integrate $f(x, y)$ over the region R. Under the transformation $x = g(u, v)$, $y = h(u, v)$ the region becomes S and the integral becomes,

$$\iint_R f(x, y)\, dA = \iint_S f(g(u, v), h(u, v)) \left| \frac{\partial (x, y)}{\partial (u, v)} \right| d\bar{A}$$

1. Show that when changing to polar coordinates we have $dA = r dr d\theta$

2. Evaluate $\int\int_R x + y dA$ where R is the trapezoidal region with vertices given by $(0, 0)$, $(5, 0)$, $(\frac{5}{2}, \frac{5}{2})$, and $(\frac{5}{2}, -\frac{5}{2})$ using the transformation $x = 2u + 3v$ and $y = 2u - 3v$.

3. Evaluate $\int\int_R x^2 - xy + y^2 dA$ where R is the ellipse given by $x^2 - xy + y^2 \leq 2$ and using the transformation $x = \sqrt{2}u - \sqrt{\frac{2}{3}}v$, $y = \sqrt{2}u + \sqrt{\frac{2}{3}}v$.

4. Verify that $dV = \rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi$ when using spherical coordinates.