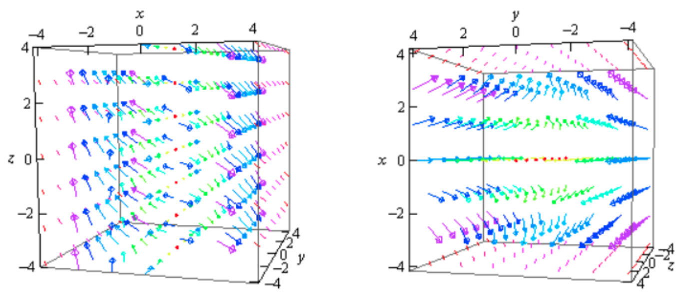
Vector Field in R^3 is represented by a vector whose components are functions:

$$\mathbf{F}(x, y, z) = \langle \mathbf{F}_1(x, y, z), \mathbf{F}_2(x, y, z), \mathbf{F}_3(x, y, z) \rangle$$

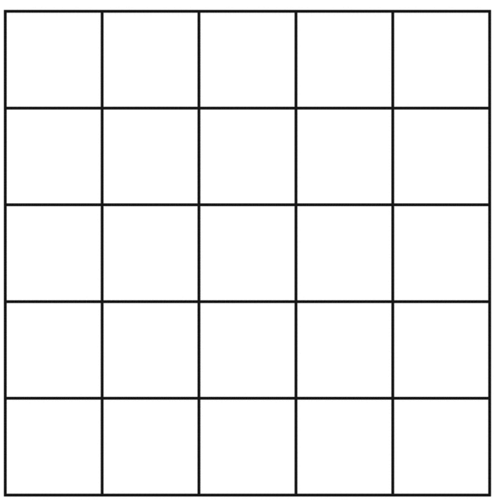
$$\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$$

It is not practical to draw a vector field in R^3, so we use a computer program to do it for us:

$\vec{F}(x, y, z) = 2x\,\vec{i} - 2y\,\vec{j} - 2x\,\vec{k}$



1. Sketch the vector field for $\vec{F}(x, y) = -y\,\vec{i} + x\,\vec{j}$



$$\nabla f = \langle f_x, f_y, f_z \rangle$$

This is a vector field and often called the gradient vector field

2. Find the gradient vector field of:
   a. $f(x, y) = x^2 \sin(5y)$
   b. $f(x, y, z) = ze^{-xy}$

Divergence of a vector field $\mathbf{F}=\langle F_1, F_2, F_3 \rangle$

$$\text{div}(\mathbf{F}) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial x} + \frac{\partial F_3}{\partial x}$$

Often write as a dot product:

$\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$

$$\nabla \bullet \mathbf{F} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \bullet \langle F_1, F_2, F_3 \rangle = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial x} + \frac{\partial F_3}{\partial x}$$

Properties:

1. div(**F**+**G**)=div(**F**)+div(**G**)

2. div(c**F**)=$c$div(**F**)

What is divergence?

Example: consider a gas with a velocity vector field given by **F.**

If div(**F**)>0 at a point P, then an outflow of gas occurs, expanding
If div(**F**)<0 at a point P, then the gas is compressing toward P
If div(**F**)=0 the gas is neither compressing nor expanding

3. Evaluate the divergence of $\mathbf{F}=\langle e^{xy}, xy, z^4 \rangle$ at $P = (1, 0, 2)$

Curl of a vector field **F**=$\langle F_{1,} F_{2,} F_{3} \rangle$

$$\mathrm{curl}(\mathbf{F}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\mathbf{i} - \left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right)\mathbf{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\mathbf{k}$$

curl(**F**)=$\nabla \times$ **F**

Properties:

3. curl(**F**+**G**)=curl(**F**)+curl(**G**)

4. curl(c**F**)=c curl(**F**)

What is curl?

Curl tells us about the rotation.

4. Calculate the curl of **F**=$\langle xy, e^{x}, y + z \rangle$