

Line Integrals  
Multivariable Calculus

[https://www.youtube.com/watch?v=WA5\\_a3C2iqY&list=PLHXZ9OQGMqxfW0GMqeUE1bLKaYo6kbHa&index=3&t=612s](https://www.youtube.com/watch?v=WA5_a3C2iqY&list=PLHXZ9OQGMqxfW0GMqeUE1bLKaYo6kbHa&index=3&t=612s)

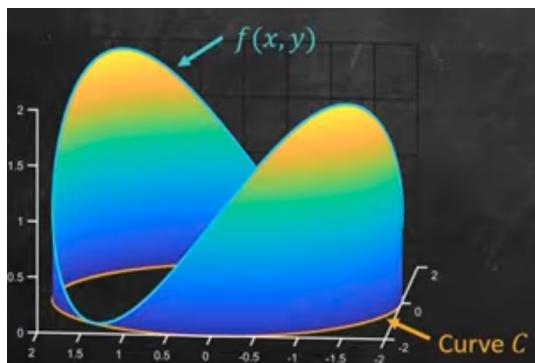
$$\int_C f(x, y) \, ds = \int_a^b f(h(t), g(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Curve	Parametric Equations	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Ellipse)	Counter-Clockwise $x = a \cos(t)$ $y = b \sin(t)$ $0 \leq t \leq 2\pi$	Clockwise $x = a \cos(t)$ $y = -b \sin(t)$ $0 \leq t \leq 2\pi$
$x^2 + y^2 = r^2$ (Circle)	Counter-Clockwise $x = r \cos(t)$ $y = r \sin(t)$ $0 \leq t \leq 2\pi$	Clockwise $x = r \cos(t)$ $y = -r \sin(t)$ $0 \leq t \leq 2\pi$
$y = f(x)$	$x = t$ $y = f(t)$	
$x = g(y)$	$x = g(t)$ $y = t$	

$$\vec{r}(t) = (1-t) \langle x_0, y_0, z_0 \rangle + t \langle x_1, y_1, z_1 \rangle, \quad 0 \leq t \leq 1$$

Line Segment From or  
 $(x_0, y_0, z_0)$  to  $x = (1-t)x_0 + t x_1$   
 $(x_1, y_1, z_1)$   $y = (1-t)y_0 + t y_1, \quad 0 \leq t \leq 1$   
 $z = (1-t)z_0 + t z_1$

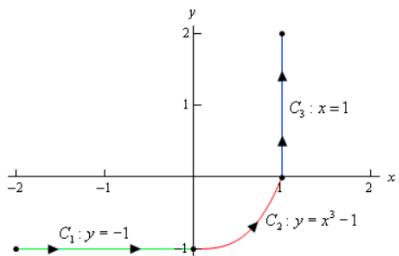
- Calculate the line integral of  $f(x, y) = \frac{x^2+y^2}{4} + \frac{xy}{2}$  above the circle of radius 2 centered at the origin.



Line Integrals  
Multivariable Calculus

2. Evaluate  $\int_C xy^4 ds$  where  $C$  is the right half of the circle  $x^2 + y^2 = 16$  traced out in a counter-clockwise direction.

3. Evaluate  $\int_C 4x^3 ds$  where  $C$  is the curve shown below.



Line Integrals  
Multivariable Calculus

4. Evaluate  $\int_C 4x^3 ds$  where  $C$  is the line segment from  $(-2, -1)$  to  $(1, 2)$

5. Evaluate  $\int_C 4x^3 ds$  where  $C$  is the line segment from  $(1, 2)$  to  $(-2, -1)$