The **line integral of f with respect to x** is,

$$\int_C f(x,y)\,dx = \int_a^b f(x(t),y(t))\,x'(t)\,dt$$

1. Evaluate $\int_C \sin(\pi y)dy + yx^2 dx$ where C is the line segment from $(0,2)$ to $(1,4)$

$\vec{r}(t) = (1-t)\langle 0,2\rangle + t\langle 1,4\rangle \quad 0 \le t \le 1$

$x(t) = t \qquad y(t) = 2(1-t) + 4t = 2 + 2t$

$\int_C \sin(\pi y)\,dy + yx^2\,dx = \int \sin(\pi y)\,dy + yx^2\,dx$

$= \int_0^1 \sin[\pi(2+2t)]\,2\,dt + (2+2t)t^2(1)dt$

$= \boxed{7/6}$

2. Evaluate $\int_C ydx + xdy + zdz$ where C is given by $x = \cos t$, $y = \sin t$, $z = t^2$, $0 \le t \le 2\pi$.

$\int y\,dx + \int x\,dy + \int z\,dz$

$\int_0^{2\pi} \sin t(-\sin t)\,dt + \int_0^{2\pi} \cos t(\cos t)\,dt + \int_0^{2\pi} t^2(2t)\,dt$

$= \boxed{8\pi^4}$