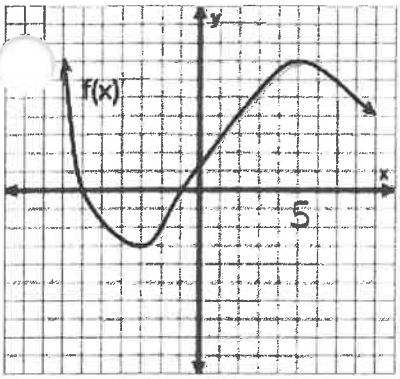


1. Place your pencil on the graph
2. Trace along the graph...STOP when you are about to hit the point where $x = 5$, but you don't actually hit that point (get infinitely close!)
3. What are you about to hit? (5, 7)

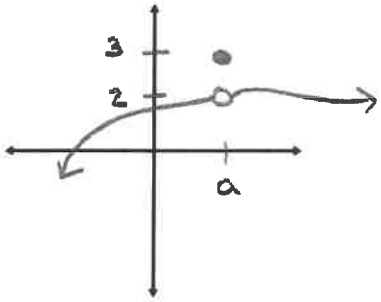
   $\lim_{x \to 5} f(x) = 7$

   ☆ notation

   $\lim_{x \to \#}$ function = y-value
4. What is $f(5) =$ 7

5. Will $\lim_{x \to a} f(x)$ always be the same value as $f(a)$? Can you draw a sketch of a graph in which they are NOT equal?

   No!



$\lim_{x \to a} f(x) = 2$ but $f(a) = 3$

limit is what you are about to hit when infinitely close to x-value

function value is the actual point @ x-value

3 ways to solve a limit:

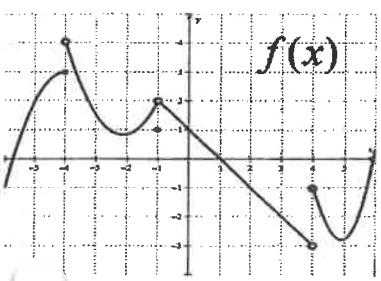
Tabular

Find $\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$

x approaches 1 from the left ⟹ ⟸ x approaches 1 from the right

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	1.9	1.99	1.999	und	2.001	2.01	2.1

Graphically



$\lim_{x \to 2} f(x) = -1$

$\lim_{x \to 0} f(x) = 1$

$\lim_{x \to -1} f(x) = 2$

What do you think the $\lim_{x \to -4} f(x) = ?$

und! approaching 2 different values from left & right

Algebraically

b/c $\lim_{x \to a} f(x) = f(a)$ when

*only works if the function is continuous or can manipulate function to evaluate

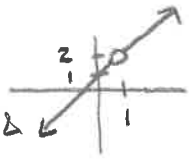
$\lim_{x \to -1} \sqrt{5x^2+4} = \sqrt{5(-1)^2+4} = 3$

continuous @ $x = -1$

$\lim_{x \to 1} \frac{x^2-1}{x-1} = \lim_{x \to 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x \to 1} x+1$

*hole @ $x = 1$

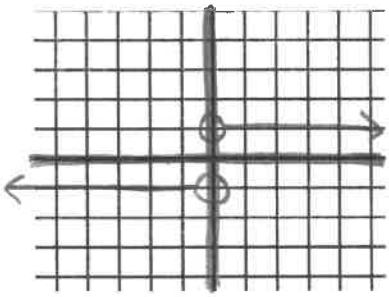
we are ignoring it by canceling $x-1$ & evaluating @ $x = 1$



$= 1 + 1$

$= 2$

One Sided Limits



$f(x) = \frac{x}{|x|}$

$\lim_{x \to 0^-} f(x) = -1$ approach from left

$\lim_{x \to 0^+} f(x) = 1$ approach from right

Properties of Limits:

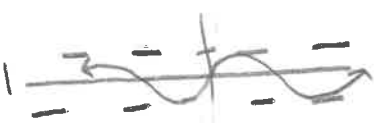
Limits of Sums, Differences, Products, Powers, Roots, and Quotients:

1. **Sum Rule**
   $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. **Difference Rule**
   $\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. **Product Rule**
   $\lim_{x \to a}[f(x) * g(x)] = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$
4. **Constant Multiple Rule**
   $\lim_{x \to a}[k * f(x)] = k \lim_{x \to a} f(x)$
5. **Power Rule**
   $\lim_{x \to a}[f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$
6. **Quotient Rule**
   $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

Sandwich Theorem

Show that $\lim_{x \to 0} x^2 \sin x = 0$

sinx bounded between −1 and 1



$-1 \le \sin x \le 1$

manipulate 3 components to be function in question

$-1(x^2) \le x^2 \sin x \le 1(x^2)$

$-x^2 \le x^2 \sin x \le x^2$

$\lim_{x \to 0}(-x^2) \le \lim_{x \to 0} x^2 \sin x \le \lim_{x \to 0} x^2$

$0 \le \lim_{x \to 0} x^2 \sin x \le 0$

so $\lim_{x \to 0} x^2 \sin x = 0$