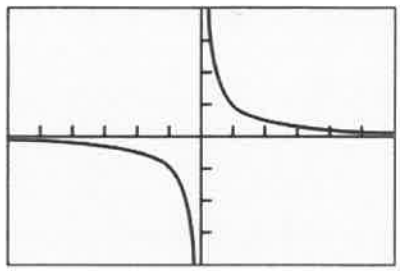
Horizontal Asymptote	The line $y = b$ when: $\lim_{x\to\infty} f(x) = b$ $\lim_{x\to-\infty} f(x) = b$

1. Use the graph below to determine:



HA $y = 0$

a. $\lim_{x\to\infty} f(x) = 0$

b. $\lim_{x\to-\infty} f(x) = 0$

2. Find the $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ of:

a. $f(x) = \frac{3(x+2)^2}{4-x^2}$ HA: $y = -3$

$\lim_{x\to\infty} f(x) = -3$

$\lim_{x\to-\infty} f(x) = -3$

b. $f(x) = \frac{3x^3-x+1}{x+3}$ HA → slant/oblique

$\frac{3x^3-x+1}{x+3} \approx \frac{3x^3}{x} = 3x^2$

will behave like $y = 3x^2$ when $x \to \infty$ & $x \to -\infty$

$\lim_{x\to\infty} f(x) = \infty$

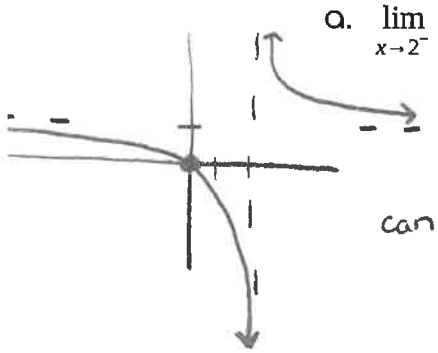
$\lim_{x\to-\infty} f(x) = \infty$

★ graph on ★ calc to prove

Vertical Asymptote	The line $x = a$ when: $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$

3. Find the following:

a. $\lim_{x\to 2^-} \frac{x}{x-2} = -\infty$

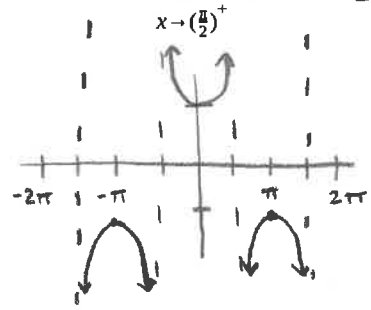


can graph f(x) or use tabular method

x	1	1.5
f(x)	$-\frac{1}{2}$	-3

★ know VA so will be either ∞ or $-\infty$ can see from table will be $-\infty$

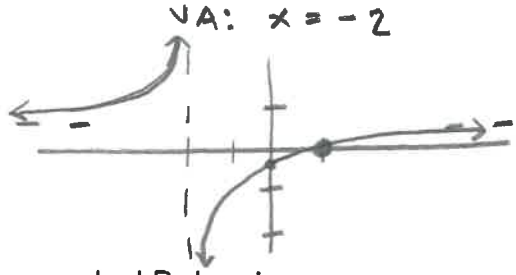
b. $\lim_{x\to(\frac{\pi}{2})^+} \sec x = -\infty$



4. Find the vertical asymptotes of the graph of $f(x)$: Describe the behavior of $f(x)$ to the left and right of each vertical asymptote.

   a. $f(x) = \frac{x-1}{2x+4}$

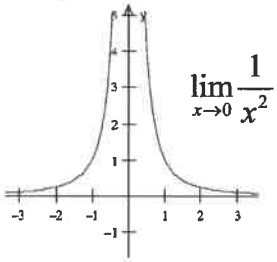
VA: $x = -2$



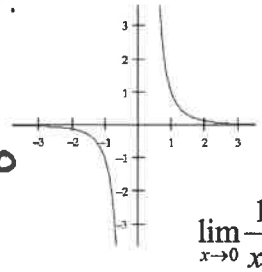
$\lim_{x \to -2^-} f(x) = \infty$

$\lim_{x \to -2^+} f(x) = -\infty$

Unbounded Behavior:
behavior:

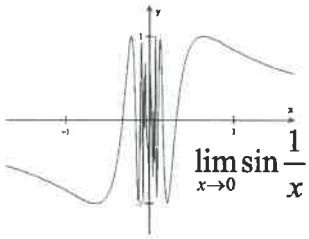


$\lim_{x \to 0} \frac{1}{x^2} = \infty$



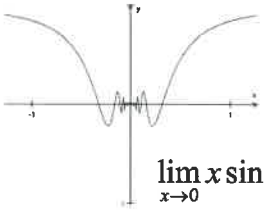
$\lim_{x \to 0} \frac{1}{x^3} = \text{DNE}$

Oscillating Behavior:



$\lim_{x \to 0} \sin\frac{1}{x} = \text{DNE}$

*Exception to oscillating



$\lim_{x \to 0} x\sin\frac{1}{x} = 0$

5. Sketch a graph of a function $y = f(x)$ that satisfies the stated conditions. Include any asymptotes.

$\lim_{x \to 2} f(x) = -1$ $\quad$ $\lim_{x \to 4^+} f(x) = -\infty$ $\quad$ $\lim_{x \to 4^-} f(x) = \infty$

$\lim_{x \to \infty} f(x) = \infty$ $\quad$ $\lim_{x \to -\infty} f(x) = 2$