

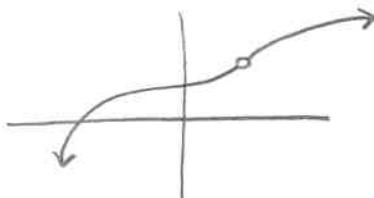
Continuity:

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

in other words:  
 the limit exists  
 AND the limit  
 equals the function  
 value

4 types of Discontinuity:

1. Removable



$$\text{ex: } f(x) = \frac{x-3}{(x-3)(x+3)}$$

2. Jump

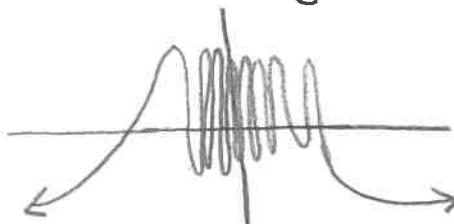


ex) piecewise

3. Infinite



4. Oscillating


 1. Find the points of discontinuity in  $f(x) = \frac{x^2-2x-3}{x^2+x-12}$  and determine each type of discontinuity.

$$f(x) = \frac{(x-3)(x+1)}{(x-3)(x+4)}$$

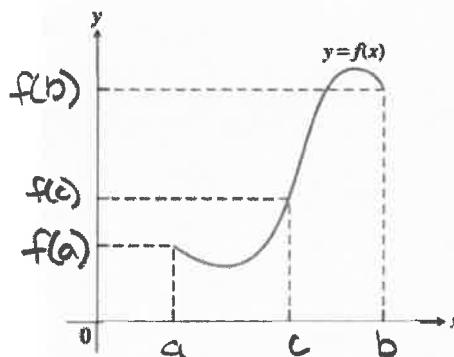
$$= \frac{x+1}{x+4}, \quad x \neq 3$$

$f(x)$  discontinuous  $\circlearrowleft x = 3$  due to  
 a removable discontinuity

$\circlearrowleft x = -4$  due to  
 an infinite discontinuity

### Intermediate Value Theorem for Continuous Functions:

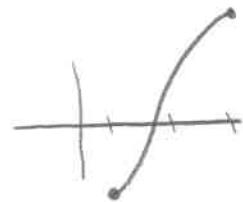
A function  $y = f(x)$  that is continuous on a closed interval  $[a, b]$  takes on every value between  $f(a)$  and  $f(b)$ . In other words, if  $y_0$  is between  $f(a)$  and  $f(b)$ , then  $y_0 = f(c)$  for some  $c$  in  $[a, b]$ .



2. Let  $f(x) = x^2 + 2x - 8$ . Prove that there is an  $x$ -intercept from  $[1, 3]$ .

$$\begin{aligned} f(1) &= 1^2 + 2 - 8 \\ &= -5 \end{aligned}$$

$$\begin{aligned} f(3) &= 3^2 + 6 - 8 \\ &= 7 \end{aligned}$$



\*since continuous must cross x-axis

Since  $f(x)$  is a continuous function on

$[1, 3]$  and  $f(1) = -5$  and  $f(3) = 7$  by

the INT there must exist a  $c$

where  $1 < c < 3$  such that  $f(c) = 0$

since  $f(1) < f(c) < f(3)$ ,  $-5 < 0 < 7$

3. Find each point of discontinuity for the function below. Then if there are any, determine if the discontinuities are removable.

\*both pieces of  $f(x) = \begin{cases} -2x, & x \leq 2 \\ x^2 - 4x + 4, & x > 2 \end{cases}$  are continuous

so opportunity for discontinuity

@  $x = 2$

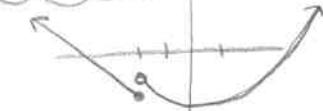
$$\begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= -2(2) \\ &= -4 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 2^+} f(x) &= 2^2 - 4(2) + 4 \\ &= -3 \end{aligned}$$

$$\begin{aligned} f(2) &= -2(2) \\ &= -4 \end{aligned}$$

def of continuity

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$



The function is discontinuous @  $x = 2$  due to a jump discontinuity  
 The  $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$

4. Find the constant  $a$ , such that the function is continuous on the entire number line.

$$f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$$

$$\lim_{x \rightarrow 2^-} f(x) = 2^3 = 8$$

$$\lim_{x \rightarrow 2^+} f(x) = a(2)^2 = 4a$$

$$f(2) = 2^3 = 8$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$$

$$8 = 4a$$

$$2 = a$$