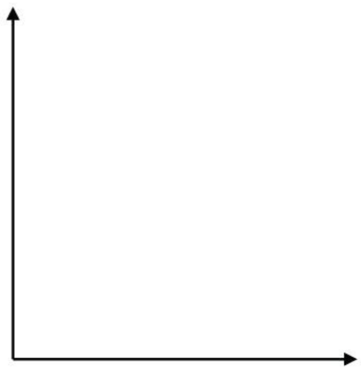
AB Calculus

2.4 Rates of Change and Tangent Lines

1. Find the average rate of change of $f(x) = \cos t$ over the interval $[0, \pi]$

Instantaneous Rate of Change



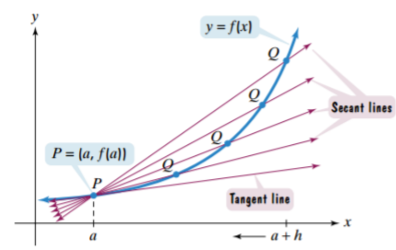
Slope of the tangent Line to a Curve at a Point

The slope of the tangent line to the graph of a function $y = f(x)$ at $(a, f(a))$ is given by:

Provided that this limit exists. This limit describes:

- The slope of the graph of f at $(a, f(a))$
- The instantaneous rate of change of f with respect to x at a

Want secant line to be as close to tangent line as possible:



Example:

1. Find the slope of the tangent line to the graph of $f(x) = x^2 + x$ at $(2, 6)$.

2. Find the slope of the tangent line to the graph $f(x) = \frac{2}{x}$ at (1, 2). Then find the tangent line at the given point.

Limit Definition of the Slope (derivative)	Alternative Definition
$m_{tan} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ = slope at a	$m_{tan} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

3. The equations below, if evaluated, will give the slope of a tangent line at some exact x −value on a function. Determine what that x −value is and the function.

a. $\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$

b. $\lim_{h\to 0} \frac{(1+h)^3-2-(-1)}{h}$

c. $\lim_{x\to 3} \frac{5x^2-45}{x-3}$

d. $\lim_{x\to \frac{1}{3}} \frac{\ln x-\ln(1/3)}{x-1/3}$