Theorem 2.5 Properties of Angle Congruence

Reflexive Property of Congruence

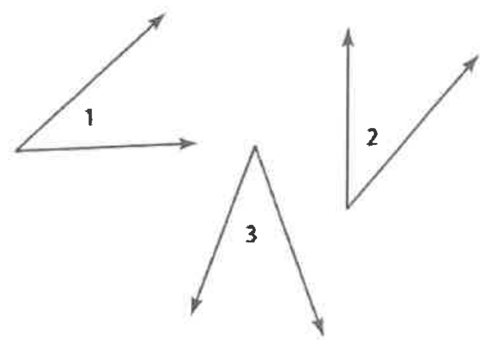
$\angle 1 \cong \angle 1$

Symmetric Property of Congruence

If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property of Congruence

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.



1. Prove the transitive property of angle congruence:

Given: $\angle 1 \cong \angle 2$ *hypothesis
$\angle 2 \cong \angle 3$

Prove: $\angle 1 \cong \angle 3$ *conclusion

Statements	Reasons
1. $\angle 1 \cong \angle 2$ $\angle 2 \cong \angle 3$	1. Given
2. $m\angle 1 = m\angle 2$	2. Def of congruent angles
3. $m\angle 2 = m\angle 3$	3. Def of $\cong$ $\angle$s
4. $m\angle 1 = m\angle 3$	4. Transitive Prop of Equality
5. $\angle 1 \cong \angle 3$	5. Def of $\cong$ $\angle$s

2. Given:

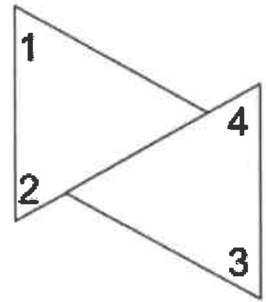
$m\angle 3 = 40\ ^\circ$

$\angle 1 \cong \angle 2$

$\angle 2 \cong \angle 3$

Prove:

$m\angle 1 = 40\ ^\circ$

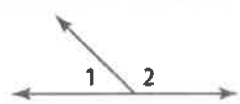


Statements	Reasons
1. m<3 = 40° <2 ≅ <3 <1 ≅ <2	1. Given
2. <1 ≅ <3	2. Transitive Prop of Angle Congruence
3. m<1 = m<3	3. Def of ≅ <s
4. m<1 = 40°	4. Substitution Prop of equality

Theorems

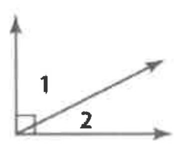
2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.

Example $m\angle 1 + m\angle 2 = 180$



2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

Example $m\angle 1 + m\angle 2 = 90$

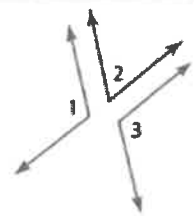


Supplement Theorem also called Linear Pair Postulate

Theorems

2.6 Congruent Supplements Theorem

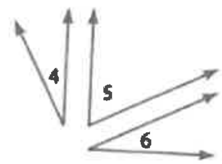
If two angles are supplementary to the same angle (or to congruent angles) then they are congruent.



Example If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.

2.7 Congruent Complements Theorem

If two angles are complementary to the same angle then the two angles are congruent.



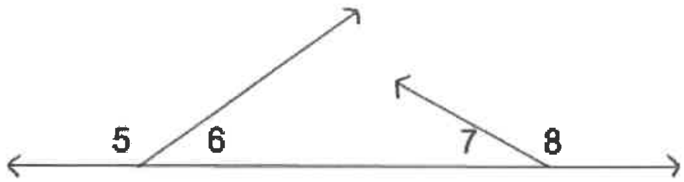
Example If $m\angle 4 + m\angle 5 = 90$ and $m\angle 5 + m\angle 6 = 90$, then $\angle 4 \cong \angle 6$.

3. Prove the Congruent Supplements Theorem: If two angles are supplementary to the same angle (or to congruent angles) then they are congruent.

Given: $\angle 1$ and $\angle 2$ are supplements
$\angle 3$ and $\angle 4$ are supplements
$\angle 1 \cong \angle 4$
Prove: $\angle 2 \cong \angle 3$

Statements	Reasons
1. <1 and <2 are supplements <3 and <4 are supplements <1 ≅ <4	1. Given
2. $m<1 + m<2 = 180°$ $m<3 + m<4 = 180°$	2. Def of Supplementary Angles
3. $m<1 + m<2 = m<3 + m<4$	3. Transitive Prop of Equality
4. $m<1 = m<4$	4. Def of Cong Angles
5. $m<1 + m<2 = m<3 + m<1$	5. Substitution Prop of Eq
6. $m<2 = m<3$	6. Subtraction Prop of Eq
7. <2 ≅ <3	7. Def of ≅ <s

4. Use the Transitive Property of Equality and the Supplements Theorem (Linear Pair Postulate) to find the $m\angle 7$, give $m\angle 8 = 125$ °



$m\angle 7 + m\angle 8 = 180$

$m\angle 7 + 125 = 180$

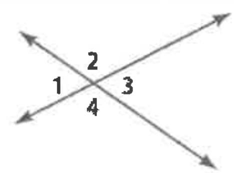
$m\angle 7 = 55°$

Theorem 2.8 Vertical Angles Theorem

If two angles are vertical angles, then they are congruent.

Abbreviation *Vert. ∡ are ≅.*

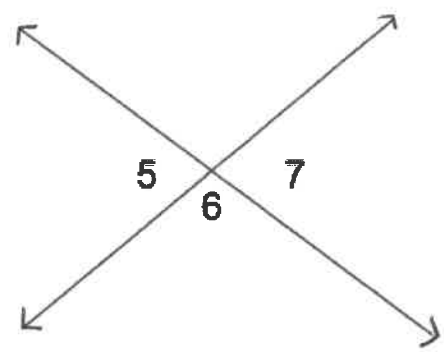
Example $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$



5. Prove the Vertical Angles Theorem:

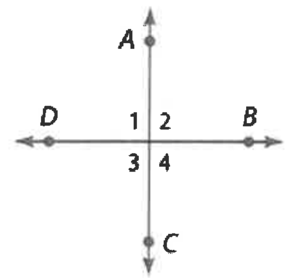
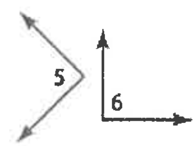
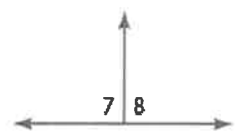
Given: $\angle 5$ and $\angle 6$ are a linear pair
$\angle 6$ and $\angle 7$ are a linear pair

Prove: $\angle 5 \cong \angle 7$



Statements	Reasons
1. <5 and <6 are a linear pair <6 and <7 are a linear pair	1. Given
2. <5 and <6 are supp <6 and <7 are supp	2. Supplement Thm
3. <5 ≅ <7	3. Congruent Supplements Thm

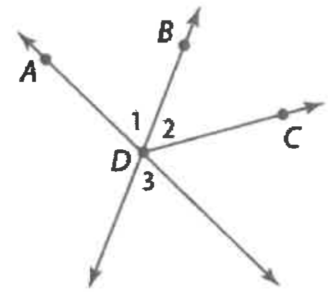
Theorems Right Angle Theorems

Theorem	Example
2.9 Perpendicular lines intersect to form four right angles. **Example** If $\overrightarrow{AC} \perp \overrightarrow{DB}$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\angle$s.	
2.10 All right angles are congruent. **Example** If $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\angle$s, then $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.	
2.11 Perpendicular lines form congruent adjacent angles. **Example** If $\overrightarrow{AC} \perp \overrightarrow{DB}$, then $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 4$, $\angle 3 \cong \angle 4$, and $\angle 1 \cong \angle 3$.	
2.12 If two angles are congruent and supplementary, then each angle is a right angle. **Example** If $\angle 5 \cong \angle 6$ and $\angle 5$ is suppl. to $\angle 6$, then $\angle 5$ and $\angle 6$ are rt. $\angle$s.	
2.13 If two congruent angles form a linear pair, then they are right angles. **Example** If $\angle 7$ and $\angle 8$ form a linear pair, then $\angle 7$ and $\angle 8$ are rt. $\angle$s.	

6. Prove that if $\overrightarrow{DB}$ bisects $\angle ADC$, then $\angle 2 \cong \angle 3$

Given: $\overrightarrow{DB}$ bisects $\angle ADC$

Prove: $\angle 2 \cong \angle 3$



Statements	Reasons
1. $\overrightarrow{DB}$ bisects $\angle ADC$	1. Given
2. $\angle 1 \cong \angle 2$	2. Def of Bisector
3. $\angle 1$ & $\angle 3$ are vertical angles	3. Def of vertical Angles
4. $\angle 1 \cong \angle 3$	4. Vertical Angles Thm
5. $\angle 2 \cong \angle 3$	5. Transitive Prop of Congruence
6.	6.