2.8 Functions Operations and Compositions
Honors Algebra 2 with Trig

1. Given the functions $f(x) = x + 2$ and $g(x) = x^2 - 4$. Find each of the following and state the domain for each new function.

a. $(f+g)(x) = f(x) + g(x)$

$= (x+2) + (x^2-4)$

$= \boxed{x^2 + x - 2}$ Domain $\mathbb{R}$

b. $(f-g)(x) = f(x) - g(x)$

$= (x+2) - (x^2-4)$

$= \boxed{-x^2 + x + 6}$ Domain $\mathbb{R}$

c. $(fg)(x) = f(x)g(x)$

$= (x+2)(x^2-4)$

$= \boxed{x^3 + 2x^2 - 4x - 8}$ Domain $\mathbb{R}$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \frac{x+2}{x^2-4}$

$= \frac{x+2}{(x+2)(x-2)} = \boxed{\frac{1}{x-2}}$ Domain $\{\mathbb{R} \mid x \neq -2, 2\}$ or $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

2. Given the functions $f(x) = \sqrt{x+1}$ and $g(x) = x^2 - 4$. Find each of the following and state the domain for each new function

a. $(f \circ g)(x) = f(g(x))$

$= \sqrt{(x^2-4)+1}$

$= \boxed{\sqrt{x^2-3}}$ domain $[\sqrt{3}, \infty)$

Domain $f(g(x))$

$x^2 - 3 \geq 0$

$x^2 \geq 3$

$|x| \geq \sqrt{3}$

$x \leq -\sqrt{3}$ or $x \geq \sqrt{3}$

$g(x)$ domain $\mathbb{R}$

$f(x)$ domain $x \geq -1$

b. $(g \circ f)(x) = g(f(x))$

$= (\sqrt{x+1})^2 - 4$

$= x + 1 - 4$

$= \boxed{x - 3}$ domain $[-1, \infty)$

domain $g(x)$ $\mathbb{R}$

domain $f(x)$ $x \geq -1$

domain $g(f(x))$ $\mathbb{R}$

3. Given the functions $f(x) = \sqrt{x}$ and $g(x) = x - 3$. Find each of the following and state the domain for each new function.

a. $(f \circ g)(x)$

$f(g(x)) = \boxed{\sqrt{x-3}}$ domain $[3, \infty)$

domain $f(x)$ $x \geq 0$

domain $g(x)$ $\mathbb{R}$

domain $f(g(x))$ $x \geq 3$

b. $(g \circ f)(x)$

$g(f(x)) = (\sqrt{x}) - 3$

$= \sqrt{x} - 3$ domain $[0, \infty)$

domain $f(x)$ $x \geq 0$

domain $g(x)$ $\mathbb{R}$

domain $g(f(x))$ $x \geq 0$

*can you come up w/ more difficult answer?

4. Express the given function h as a composition of two functions f and g so that $h(x) = (f \circ g)(x)$.

a. $h(x) = (2x-5)^5$

$f(x) = x^5$

$g(x) = 2x-5$

b. $h(x) = \sqrt{5x^2+3}$

$f(x) = \sqrt{x}$

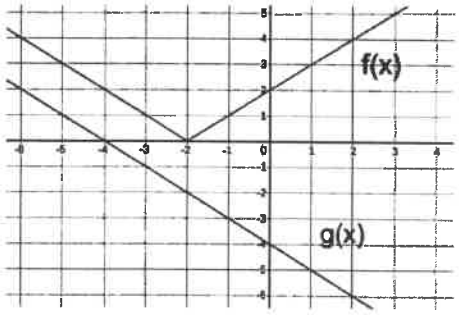
$g(x) = 5x^2+3$

c. $h(x) = |3x-4|$

$f(x) = |x|$

$g(x) = 3x-4$

5. Use the graphs of f and g below to evaluate each function.



a. $(f+g)(-4) = \boxed{2}$

$= f(-4) + g(-4)$

$= 2 + 0$

b. $(f-g)(2) = \boxed{10}$

$= f(2) - g(2)$

$= 4 - (-6)$

c. $\left(\frac{f}{g}\right)(-5)$

$= \frac{f(-5)}{g(-5)} = \frac{3}{1} = \boxed{3}$

d. $(f \circ g)(1)$

$= f(g(1))$

$= f(-5)$

$= \boxed{3}$

e. $(g \circ f)(-1)$

$= g(f(-1))$

$= g(1)$

$= \boxed{-5}$

6. Let $f(x) = 2x^2 - 3x$. Find and simplify the expression for the difference quotient $\frac{f(x+h)-f(x)}{h}$

$$= \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \frac{4xh + 2h^2 - 3h}{h} = \boxed{4x + 2h - 3}$$