2017 AMC 12B Problems

WORK IN PROGRESS

2017 AMC 12B (Answer Key)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.
3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will *require* the use of a calculator).
4. Figures are not necessarily drawn to scale.
5. You will have **75 minutes** working time to complete the test.

1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25

Contents

Problem 1

Kymbrea's comic book collection currently has 30 comic books in it, and she is adding to her collection at the rate of 2 comic books per month. LaShawn's collection currently has 10 comic books in it, and he is adding to his collection at the rate of 6 comic books per month. After how many months will LaShawn's collection have twice as many comic books as Kymbrea's?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Solution

Problem 2

Real numbers x, y, and z satify the inequalities $0 < x < 1$, $-1 < y < 0$, and $1 < z < 2$. Which of the following numbers is necessarily positive?

$\textbf{(A)}\ y+x^2 \qquad \textbf{(B)}\ y+xz \qquad \textbf{(C)}\ y+y^2 \qquad \textbf{(D)}\ y+2y^2 \qquad \textbf{(E)}\ y+z$

Solution

Problem 3

Supposed that x and y are nonzero real numbers such that $\frac{3x+y}{x-3y}=-2$. What is the value of $\frac{x+3y}{3x-y}$?

$\textbf{(A)}\ -3 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Solution

Problem 4

Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

$\textbf{(A)}\ 2.0 \qquad \textbf{(B)}\ 2.2 \qquad \textbf{(C)}\ 2.8 \qquad \textbf{(D)}\ 3.4 \qquad \textbf{(E)}\ 4.4$

Solution

Problem 5

The data set $[6, 19, 33, 33, 39, 41, 41, 43, 51, 57]$ has median $Q_2 = 40$, first quartile $Q_1 = 33$, and third quartile $Q_3 = 43$. An outlier in a data set is a value that is more than 1.5 times the interquartile range below the first quartile (Q_1) or more than 1.5 times the interquartile range above the third quartile (Q_3), where the interquartile range is defined as $Q_3 - Q_1$. How many outliers does this data set have?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Solution

Problem 6

The circle having $(0,0)$ and $(8,6)$ as the endpoints of a diameter intersects the x-axis at a second point. What is the x-coordinate of this point?

$\textbf{(A)}\ 4\sqrt{2} \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 5\sqrt{2} \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 6\sqrt{2}$

Solution

Problem 7

The functions $\sin(x)$ and $\cos(x)$ are periodic with least period 2π. What is the least period of the function $\cos(\sin(x))$?

$\textbf{(A)}\ \frac{\pi}{2} \qquad \textbf{(B)}\ \pi \qquad \textbf{(C)}\ 2\pi \qquad \textbf{(D)}\ 4\pi \qquad \textbf{(E)}$ It's not periodic.

Solution

Problem 8

The ratio of the short side of a certain rectangle to the long side is equal to the ratio of the long side to the diagonal. What is the square of the ratio of the short side to the long side of this rectangle?

$\textbf{(A)}\ \frac{\sqrt{3}-1}{2} \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{\sqrt{5}-1}{2} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{2} \qquad \textbf{(E)}\ \frac{\sqrt{6}-1}{2}$

Solution

Problem 9

A circle has center $(-10,-4)$ and radius 13. Another circle has center $(3,9)$ and radius $\sqrt{65}$. The line passing through the two points of intersection of the two circles has equation $x+y=c$. What is C?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 3\sqrt{3} \qquad \textbf{(C)}\ 4\sqrt{2} \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ \frac{13}{2}$

Solution

Problem 10

At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

$\textbf{(A)}\ 10\% \qquad \textbf{(B)}\ 12\% \qquad \textbf{(C)}\ 20\% \qquad \textbf{(D)}\ 25\% \qquad \textbf{(E)}\ 33\frac{1}{3}\%$

Solution

Problem 11

Call a positive integer $monotonous$ if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

$\textbf{(A)}\ 1024 \qquad \textbf{(B)}\ 1524 \qquad \textbf{(C)}\ 1533 \qquad \textbf{(D)}\ 1536 \qquad \textbf{(E)}\ 2048$

Solution

Problem 12

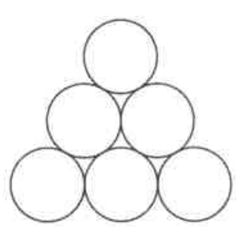
What is the sum of the roots of $z^{12}=64$ that have a positive real part?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ \sqrt{2}+2\sqrt{3} \qquad \textbf{(D)}\ 2\sqrt{2}+\sqrt{6} \qquad \textbf{(E)}\ (1+\sqrt{3})+(1+\sqrt{3})i$

Solution

Problem 13

In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 15$

Solution

Problem 14

An ice-cream novelty item consists of a cup in the shape of a 4-inch-tall frustum of a right circular cone, with a 2-inch-diameter base at the bottom and a 4-inch-diameter base at the top, packed solid with ice cream, together with a solid cone of ice cream of height 4 inches, whose base, at the bottom, is the top base of the frustum. What is the total volume of the ice cream, in cubic inches?

$\textbf{(A)}\ 8\pi \qquad \textbf{(B)}\ \frac{28\pi}{3} \qquad \textbf{(C)}\ 12\pi \qquad \textbf{(D)}\ 14\pi \qquad \textbf{(E)}\ \frac{44\pi}{3}$

Solution

Problem 15

Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

$\textbf{(A)}\ 9:1 \qquad \textbf{(B)}\ 16:1 \qquad \textbf{(C)}\ 25:1 \qquad \textbf{(D)}\ 36:1 \qquad \textbf{(E)}\ 37:1$

Solution

Problem 16

The number $21! = 51,090,942,171,709,440,000$ has over $60,000$ positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

$\textbf{(A)}\ \frac{1}{21} \qquad \textbf{(B)}\ \frac{1}{19} \qquad \textbf{(C)}\ \frac{1}{18} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{11}{21}$

Solution

Problem 17

A coin is biased in such a way that on each toss the probability of heads is $\frac{2}{3}$ and the probability of tails is $\frac{1}{3}$. The outcomes of the tosses are independent. A player has the choice of playing Game A or Game B. In Game A she tosses the coin three times and wins if all three outcomes are the same. In Game B she tosses the coin four times and wins if both the outcomes of the first and second tosses are the same and the outcomes of the third and fourth tosses are the same. How do the chances of winning Game A compare to the chances of winning Game B?

$\textbf{(A)}$ The probability of winning Game A is $\frac{4}{81}$ less than the probability of winning Game B.

$\textbf{(B)}$ The probability of winning Game A is $\frac{2}{81}$ less than the probability of winning Game B.

$\textbf{(C)}$ The probabilities are the same.

$\textbf{(D)}$ The probability of winning Game A is $\frac{2}{81}$ greater than the probability of winning Game B.

$\textbf{(E)}$ The probability of winning Game A is $\frac{4}{81}$ greater than the probability of winning Game B.

Solution

Problem 18

The diameter AB of a circle of radius 2 is extended to a point D outside the circle so that $BD = 3$. Point E is chosen so that $ED = 5$ and line ED is perpendicular to line AD. Segment AE intersects the circle at a point C between A and E. What is the area of $\triangle ABC$?

$\textbf{(A)}\ \frac{120}{37} \qquad \textbf{(B)}\ \frac{140}{39} \qquad \textbf{(C)}\ \frac{145}{39} \qquad \textbf{(D)}\ \frac{140}{37} \qquad \textbf{(E)}\ \frac{120}{31}$

Solution

Problem 19

Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 44$

Solution

Problem 20

Real numbers x and y are chosen independently and uniformly at random from the interval $(0,1)$. What is the probability that $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor$ where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r ?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{6} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Solution

Problem 21

Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

$\textbf{(A)}\ 92 \qquad \textbf{(B)}\ 94 \qquad \textbf{(C)}\ 96 \qquad \textbf{(D)}\ 98 \qquad \textbf{(E)}\ 100$

Solution

Problem 22

Abby, Bernardo, Carl, and Debra play a game in which each of them starts with four coins. The game consists of four rounds. In each round, four balls are placed in an urn---one green, one red, and two white. The players each draw a ball at random without replacement. Whoever gets the green ball gives one coin to whoever gets the red ball. What is the probability that, at the end of the fourth round, each of the players has four coins?

$\textbf{(A)}\ \frac{7}{576} \qquad \textbf{(B)}\ \frac{5}{192} \qquad \textbf{(C)}\ \frac{1}{36} \qquad \textbf{(D)}\ \frac{5}{144} \qquad \textbf{(E)}\ \frac{7}{48}$

Solution

Problem 23

The graph of $y = f(x)$, where $f(x)$ is a polynomial of degree 3, contains points $A(2,4)$, $B(3,9)$, and $C(4,16)$. Lines AB, AC, and BC intersect the graph again at points D, E, and F, respectively, and the sum of the x-coordinates of D, E, and F is 24. What is $f(0)$? $\textbf{(A)}\ -2 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ \frac{24}{5} \qquad \textbf{(E)}\ 8$

Solution

Problem 24

Quadrilateral $ABCD$ has right angles at B and C, $\triangle ABC \sim \triangle BCD$, and $AB > BC$. There is a point E in the interior of $ABCD$ such that $\triangle ABC \sim \triangle CEB$ and the area of $\triangle AED$ is 17 times the area of $\triangle CEB$. What is $\frac{AB}{BC}$?

$\textbf{(A)}\ 1+\sqrt{2} \qquad \textbf{(B)}\ 2+\sqrt{2} \qquad \textbf{(C)}\ \sqrt{17} \qquad \textbf{(D)}\ 2+\sqrt{5} \qquad \textbf{(E)}\ 1+2\sqrt{3}$

Solution

Problem 25

A set of n people participate in an online video basketball tournament. Each person may be a member of any number of 5-player teams, but no teams may have exactly the same 5 members. The site statistics show a curious fact: The average, over all subsets of size 9 of the set of n participants, of the number of complete teams whose members are among those 9 people is equal to the reciprocal of the average, over all subsets of size 8 of the set of n participants, of the number of complete teams whose members are among those 8 people. How many values n, $9 \leq n \leq 2017$, can be the number of participants?

$\textbf{(A)}\ 477 \qquad \textbf{(B)}\ 482 \qquad \textbf{(C)}\ 487 \qquad \textbf{(D)}\ 557 \qquad \textbf{(E)}\ 562$

Solution

See also

2017 AMC 12B (Problems • Answer Key • Resources)	
Preceded by **2017 AMC 12A Problems**	Followed by **2018 AMC 12A Problems**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

Category: AMC 12 Problems

2017 AMC 12B Answer Key

1. E
2. E
3. D
4. C
5. B
6. D
7. B
8. C
9. A
10. D
11. B
12. D
13. D
14. E
15. E
16. B
17. D
18. D
19. C
20. D
21. E
22. B
23. D
24. D
25. D

2017 AMC 12A Problems

NOTE: AS OF NOW A WORK IN PROGRESS (Problems are not accurate/might not be formatted correctly)

2017 AMC 12A (Answer Key)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
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1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25

Contents

Problem 1

Pablo buys popsicles for his friends. The store sells single popsicles for $1 each, 3-popsicle boxes for $2, and 5-popsicle boxes for $3. What is the greatest number of popsicles that Pablo can buy with $8?

$\textbf{(A)}\ 8\qquad\textbf{(B)}\ 11\qquad\textbf{(C)}\ 12\qquad\textbf{(D)}\ 13\qquad\textbf{(E)}\ 15$

Solution

Problem 2

The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 4\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 12$

Solution

Problem 3

Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which one of these statements necessarily follows logically?

$\textbf{(A)}$ If Lewis did not receive an A, then he got all of the multiple choice questions wrong.
$\textbf{(B)}$ If Lewis did not receive an A, then he got at least one of the multiple choice questions wrong.
$\textbf{(C)}$ If Lewis got at least one of the multiple choice questions wrong, then he did not receive an A.
$\textbf{(D)}$ If Lewis received an A, then he got all of the multiple choice questions right.
$\textbf{(E)}$ If Lewis received an A, then he got at least one of the multiple choice questions right.

Solution

Problem 4

Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

$\textbf{(A)}\ 30\%\qquad\textbf{(B)}\ 40\%\qquad\textbf{(C)}\ 50\%\qquad\textbf{(D)}\ 60\%\qquad\textbf{(E)}\ 70\%$

Solution

Problem 5

At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?

$\textbf{(A)}\ 240 \qquad \textbf{(B)}\ 245 \qquad \textbf{(C)}\ 290 \qquad \textbf{(D)}\ 480 \qquad \textbf{(E)}\ 490$

Solution

Problem 6

Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 20$

Solution

Problem 7

Define a function on the positive integers recursively by $f(1) = 2$, $f(n) = f(n-1) + 2$ if n is even, and $f(n) = f(n-2) + 2$ if n is odd and greater than 1. What is $f(2017)$?

$\textbf{(A)}\ 2017 \qquad \textbf{(B)}\ 2018 \qquad \textbf{(C)}\ 4034 \qquad \textbf{(D)}\ 4035 \qquad \textbf{(E)}\ 4036$

Solution

Problem 8

The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume 216π. What is the length AB?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 24$

Solution

Problem 9

Let S be the set of points (x,y) in the coordinate plane such that two of the three quantities 3, $x+2$, and $y-4$ are equal and the third of the three quantities is no greater than the common value. Which of the following is a correct description of S?

$\textbf{(A)}\ \text{a single point} \qquad \textbf{(B)}\ \text{two intersecting lines}$
$\textbf{(C)}\ \text{three lines whose pairwise intersections are three distinct points}$
$\textbf{(D)}\ \text{a triangle} \qquad \textbf{(E)}\ \text{three rays with a common point}$

Solution

Problem 10

Chloé chooses a real number uniformly at random from the interval $[0, 2017]$. Independently, Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloe's number?

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{2}{3} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{5}{6} \qquad \textbf{(E)}\ \frac{7}{8}$

Solution

Problem 11

Claire adds the degree measures of the interior angles of a convex polygon and arrives at a sum of 2017. She then discovers that she forgot to include one angle. What is the degree measure of the forgotten angle?

$\textbf{(A)}\ 37 \qquad \textbf{(B)}\ 63 \qquad \textbf{(C)}\ 117 \qquad \textbf{(D)}\ 143 \qquad \textbf{(E)}\ 163$

Solution

Problem 12

There are 10 horses, named Horse 1, Horse 2, . . ., Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Solution

Problem 13

Driving at a constant speed, Sharon usually takes 180 minutes to drive from her house to her mother's house. One day Sharon begins the drive at her usual speed, but after driving $\frac{1}{3}$ of the way, she hits a bad snowstorm and reduces her speed by 20 miles per hour. This time the trip takes her a total of 276 minutes. How many miles is the drive from Sharon's house to her mother's house?

$\textbf{(A)}\ 132 \qquad \textbf{(B)}\ 135 \qquad \textbf{(C)}\ 138 \qquad \textbf{(D)}\ 141 \qquad \textbf{(E)}\ 144$

Solution

Problem 14

Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 28 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 40$

Solution

Problem 15

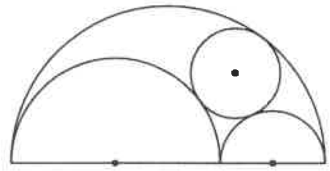
Let $f(x) = \sin{x} + 2\cos{x} + 3\tan{x}$, using radian measure for the variable x. In what interval does the smallest positive value of x for which $f(x) = 0$ lie?

$\textbf{(A)}\ (0,1) \qquad \textbf{(B)}\ (1,2) \qquad \textbf{(C)}\ (2,3) \qquad \textbf{(D)}\ (3,4) \qquad \textbf{(E)}\ (4,5)$

Solution

Problem 16

In the figure below, semicircles with centers at A and B and with radii 2 and 1, respectively, are drawn in the interior of, and sharing bases with, a semicircle with diameter JK. The two smaller semicircles are externally tangent to each other and internally tangent to the largest semicircle. A circle centered at P is drawn externally tangent to the two smaller semicircles and internally tangent to the largest semicircle. What is the radius of the circle centered at P?



$\textbf{(A)}\ \frac{3}{4} \qquad \textbf{(B)}\ \frac{6}{7} \qquad \textbf{(C)}\ \frac{1}{2}\sqrt{3} \qquad \textbf{(D)}\ \frac{5}{8}\sqrt{2} \qquad \textbf{(E)}\ \frac{11}{12}$

Solution

Problem 17

There are 24 different complex numbers z such that $z^{24} = 1$. For how many of these is z^6 a real number?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 24$

Solution

Problem 18

Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 1239 \qquad \textbf{(E)}\ 1265$

Solution

Problem 19

A square with side length x is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length y is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

$\textbf{(A)}\ \frac{12}{13} \qquad \textbf{(B)}\ \frac{35}{37} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \frac{37}{35} \qquad \textbf{(E)}\ \frac{13}{12}$

Solution

Problem 20

How many ordered pairs (a,b) such that a is a positive real number and b is an integer between 2 and 200, inclusive, satisfy the equation $(\log_b a)^{2017} = \log_b(a^{2017})$?

$\textbf{(A)}\ 198 \qquad \textbf{(B)}\ 199 \qquad \textbf{(C)}\ 398 \qquad \textbf{(D)}\ 399 \qquad \textbf{(E)}\ 597$

Solution

Problem 21

A set S is constructed as follows. To begin, $S = \{0,10\}$. Repeatedly, as long as possible, if x is an integer root of some polynomial $a_{n}x^n + a_{n-1}x^{n-1} + ... + a_{1}x + a_0$ for some $n \geq 1$, all of whose coefficients a_i are elements of S, then x is put into S. When no more elements can be added to S, how many elements does S have?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 11$

Solution

Problem 22

A square is drawn in the Cartesian coordinate plane with vertices at $(2, 2)$, $(-2, 2)$, $(-2, -2)$, $(2, -2)$. A particle starts at $(0,0)$. Every second it moves with equal probability to one of the eight lattice points (points with integer coordinates) closest to its current position, independently of its previous moves. In other words, the probability is $1/8$ that the particle will move from (x, y) to each of $(x, y + 1)$, $(x + 1, y + 1)$, $(x + 1, y)$, $(x + 1, y - 1)$, $(x, y - 1)$, $(x - 1, y - 1)$, $(x - 1, y)$, or $(x - 1, y + 1)$. The particle will eventually hit the square for the first time, either at one of the 4 corners of the square or at one of the 12 lattice points in the interior of one of the sides of the square. The probability that it will hit at a corner rather than at an interior point of a side is m/n, where m and n are relatively prime positive integers. What is $m + n$?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 39$

Solution

Problem 23

For certain real numbers a, b, and c, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

$\textbf{(A)}\ -9009 \qquad \textbf{(B)}\ -8008 \qquad \textbf{(C)}\ -7007 \qquad \textbf{(D)}\ -6006 \qquad \textbf{(E)}\ -5005$

Solution

Problem 24

Quadrilateral $ABCD$ is inscribed in circle O and has side lengths $AB=3, BC=2, CD=6$, and $DA=8$. Let X and Y be points on $\overline{BD}$ such that $\frac{DX}{BD} = \frac{1}{4}$ and $\frac{BY}{BD} = \frac{11}{36}$. Let E be the intersection of line AX and the line through Y parallel to $\overline{AD}$. Let F be the intersection of line CX and the line through E parallel to $\overline{AC}$. Let G be the point on circle O other than C that lies on line CX. What is $XF\cdot XG$?

$\textbf{(A) }17\qquad\textbf{(B) }\frac{59-5\sqrt{2}}{3}\qquad\textbf{(C) }\frac{91-12\sqrt{3}}{4}\qquad\textbf{(D) }\frac{67-10\sqrt{2}}{3}\qquad\textbf{(E) }18$

Solution

Problem 25

The vertices V of a centrally symmetric hexagon in the complex plane are given by

$$V=\left\{ \sqrt{2}i,-\sqrt{2}i, \frac{1}{\sqrt{8}}(1+i),\frac{1}{\sqrt{8}}(-1+i),\frac{1}{\sqrt{8}}(1-i),\frac{1}{\sqrt{8}}(-1-i) \right\}.$$

For each j, $1\leq j\leq 12$, an element z_j is chosen from V at random, independently of the other choices. Let $P={\prod}_{j=1}^{12}z_j$ be the product of the 12 numbers selected. What is the probability that $P=-1$?

$\textbf{(A) } \dfrac{5\cdot11}{3^{10}} \qquad \textbf{(B) } \dfrac{5^2\cdot11}{2\cdot3^{10}} \qquad \textbf{(C) } \dfrac{5\cdot11}{3^{9}} \qquad \textbf{(D) } \dfrac{5\cdot7\cdot11}{2\cdot3^{10}} \qquad \textbf{(E) } \dfrac{2^2\cdot5\cdot11}{3^{10}}$

Solution

See also

2017 AMC 12A (Problems • Answer Key • Resources)	
Preceded by **2016 AMC 12B Problems**	Followed by **2017 AMC 12B Problems**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

2017 AMC 12A Answer Key

1. D
2. C
3. B
4. A
5. B
6. B
7. B
8. D
9. E
10. C
11. D
12. B
13. B
14. C
15. D
16. B
17. D
18. D
19. D
20. E
21. D
22. E
23. C
24. A
25. E

2016 AMC 12A Problems

2016 AMC 12A (Answer Key)
Printable version: | AoPS Resources • PDF

Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.
3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will *require* the use of a calculator).
4. Figures are not necessarily drawn to scale.
5. You will have **75 minutes** working time to complete the test.

1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25

Contents

Problem 1

What is the value of $\frac{11!-10!}{9!}$?

$\textbf{(A)}\ 99\qquad\textbf{(B)}\ 100\qquad\textbf{(C)}\ 110\qquad\textbf{(D)}\ 121\qquad\textbf{(E)}\ 132$

Solution

Problem 2

For what value of x does $10^x \cdot 100^{2x} = 1000^5$?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Solution

Problem 3

The remainder can be defined for all real numbers x and y with $y \neq 0$ by

$$\text{rem}(x, y) = x - y\left\lfloor \frac{x}{y} \right\rfloor$$

where $\left\lfloor \frac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{x}{y}$. What is the value of $\text{rem}(\frac{3}{8}, -\frac{2}{5})$?

$\textbf{(A)}\ -\frac{3}{8}\qquad\textbf{(B)}\ -\frac{1}{40}\qquad\textbf{(C)}\ 0\qquad\textbf{(D)}\ \frac{3}{8}\qquad\textbf{(E)}\ \frac{31}{40}$

Solution

Problem 4

The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to x. What is the value of x?

$\textbf{(A)}\ 50\qquad\textbf{(B)}\ 60\qquad\textbf{(C)}\ 75\qquad\textbf{(D)}\ 90\qquad\textbf{(E)}\ 100$

Solution

Problem 5

Goldbach's conjecture states that every even integer greater than 2 can be written as the sum of two prime numbers (for example, $2016 = 13 + 2003$). So far, no one has been able to prove that the conjecture is true, and no one has found a counterexample to show that the conjecture is false. What would a counterexample consist of?

(A) an odd integer greater than 2 that can be written as the sum of two prime numbers
(B) an odd integer greater than 2 that cannot be written as the sum of two prime numbers
(C) an even integer greater than 2 that can be written as the sum of two numbers that are not prime
(D) an even integer greater than 2 that can be written as the sum of two prime numbers
(E) an even integer greater than 2 that cannot be written as the sum of two prime numbers

Solution

Problem 6

A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row. What is the sum of the digits of N ?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

Solution

Problem 7

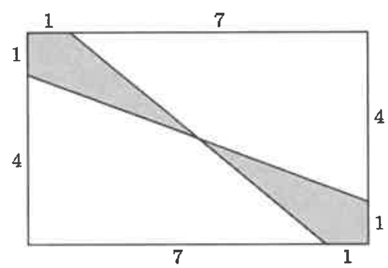
Which of these describes the graph of $x^2(x+y+1) = y^2(x+y+1)$?

(A) two parallel lines
(B) two intersecting lines
(C) three lines that all pass through a common point
(D) three lines that do not all pass through a common point
(E) a line and a parabola

Solution

Problem 8

What is the area of the shaded region of the given 8×5 rectangle?

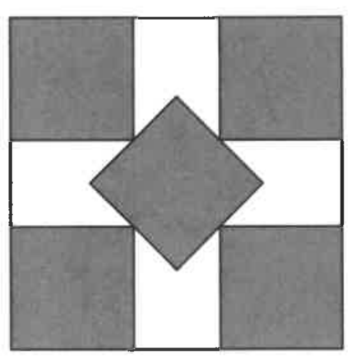


(A) $4\frac{3}{4}$ **(B)** 5 **(C)** $5\frac{1}{4}$ **(D)** $6\frac{1}{2}$ **(E)** 8

Solution

Problem 9

The five small shaded squares inside this unit square are congruent and have disjoint interiors. The midpoint of each side of the middle square coincides with one of the vertices of the other four small squares as shown. The common side length is $\frac{a-\sqrt{2}}{b}$, where a and b are positive integers. What is $a + b$?



(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Solution

Problem 10

Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Solution

Problem 11

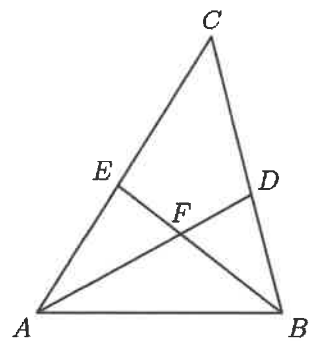
Each of the 100 students in a certain summer camp can either sing, dance, or act. Some students have more than one talent, but no student has all three talents. There are 42 students who cannot sing, 65 students who cannot dance, and 29 students who cannot act. How many students have two of these talents?

$\textbf{(A)}\ 16\qquad\textbf{(B)}\ 25\qquad\textbf{(C)}\ 36\qquad\textbf{(D)}\ 49\qquad\textbf{(E)}\ 64$

Solution

Problem 12

In $\triangle ABC$, $AB = 6$, $BC = 7$, and $CA = 8$. Point D lies on $\overline{BC}$, and $\overline{AD}$ bisects $\angle BAC$. Point E lies on $\overline{AC}$, and $\overline{BE}$ bisects $\angle ABC$. The bisectors intersect at F. What is the ratio $AF : FD$?



$\textbf{(A)}\ 3:2\qquad\textbf{(B)}\ 5:3\qquad\textbf{(C)}\ 2:1\qquad\textbf{(D)}\ 7:3\qquad\textbf{(E)}\ 5:2$

Solution

Problem 13

Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\tfrac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5)=1$ and that $P(N)$ approaches $\tfrac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \tfrac{321}{400}$?

$\textbf{(A)}\ 12\qquad\textbf{(B)}\ 14\qquad\textbf{(C)}\ 16\qquad\textbf{(D)}\ 18\qquad\textbf{(E)}\ 20$

Solution

Problem 14

Each vertex of a cube is to be labeled with an integer from 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 3\qquad\textbf{(C)}\ 6\qquad\textbf{(D)}\ 12\qquad\textbf{(E)}\ 24$

Solution

Problem 15

Circles with centers P, Q and R, having radii 1, 2 and 3, respectively, lie on the same side of line l and are tangent to l at P', Q' and R', respectively, with Q' between P' and R'. The circle with center Q is externally tangent to each of the other two circles. What is the area of triangle PQR?

$\textbf{(A)}\ 0\qquad\textbf{(B)}\ \sqrt{\frac{2}{3}}\qquad\textbf{(C)}\ 1\qquad\textbf{(D)}\ \sqrt{6}-\sqrt{2}\qquad\textbf{(E)}\ \sqrt{\frac{3}{2}}$

Solution

Problem 16

The graphs of $y=\log_3 x, y=\log_x 3, y=\log_\frac{1}{3} x,$ and $y=\log_x \dfrac{1}{3}$ are plotted on the same set of axes. How many points in the plane with positive x-coordinates lie on two or more of the graphs?

$\textbf{(A)}\ 2\qquad\textbf{(B)}\ 3\qquad\textbf{(C)}\ 4\qquad\textbf{(D)}\ 5\qquad\textbf{(E)}\ 6$

Solution

Problem 17

Let $ABCD$ be a square. Let E, F, G and H be the centers, respectively, of equilateral triangles with bases $\overline{AB}, \overline{BC}, \overline{CD},$ and $\overline{DA},$ each exterior to the square. What is the ratio of the area of square $EFGH$ to the area of square $ABCD$?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ \frac{2+\sqrt{3}}{3}\qquad\textbf{(C)}\ \sqrt{2}\qquad\textbf{(D)}\ \frac{\sqrt{2}+\sqrt{3}}{2}\qquad\textbf{(E)}\ \sqrt{3}$

Solution

Problem 18

For some positive integer $n,$ the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3.$ How many positive integer divisors does the number $81n^4$ have?

$\textbf{(A)}\ 110\qquad\textbf{(B)}\ 191\qquad\textbf{(C)}\ 261\qquad\textbf{(D)}\ 325\qquad\textbf{(E)}\ 425$

Solution

Problem 19

Jerry starts at 0 on the real number line. He tosses a fair coin 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process is $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a+b$? (For example, he succeeds if his sequence of tosses is $HTHHHHHH$.)

$\textbf{(A)}\ 69 \qquad \textbf{(B)}\ 151 \qquad \textbf{(C)}\ 257 \qquad \textbf{(D)}\ 293 \qquad \textbf{(E)}\ 313$

Solution

Problem 20

A binary operation $\diamondsuit$ has the properties that $a\,\diamondsuit\,(b\,\diamondsuit\,c) = (a\,\diamondsuit\,b)\cdot c$ and that $a\,\diamondsuit\,a = 1$ for all nonzero real numbers a, b and c. (Here the dot $\cdot$ represents the usual multiplication operation.) The solution to the equation $2016\,\diamondsuit\,(6\,\diamondsuit\,x) = 100$ can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p+q$?

$\textbf{(A)}\ 109 \qquad \textbf{(B)}\ 201 \qquad \textbf{(C)}\ 301 \qquad \textbf{(D)}\ 3049 \qquad \textbf{(E)}\ 33{,}601$

Solution

Problem 21

A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of its fourth side?

$\textbf{(A)}\ 200 \qquad \textbf{(B)}\ 200\sqrt{2} \qquad \textbf{(C)}\ 200\sqrt{3} \qquad \textbf{(D)}\ 300\sqrt{2} \qquad \textbf{(E)}\ 500$

Solution

Problem 22

How many ordered triples (x,y,z) of positive integers satisfy $\text{lcm}(x,y) = 72, \text{lcm}(x,z) = 600$ and $\text{lcm}(y,z) = 900$?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 64$

Solution

Problem 23

Three numbers in the interval $[0,1]$ are chosen independently and at random. What is the probability that the chosen numbers are the side lengths of a triangle with positive area?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{2}{3} \qquad \textbf{(E)}\ \frac{5}{6}$

Solution

Problem 24

There is a smallest positive real number a such that there exists a positive real number b such that all the roots of the polynomial x^3-ax^2+bx-a are real. In fact, for this value of a the value of b is unique. What is the value of b?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

Solution

Problem 25

Let k be a positive integer. Bernardo and Silvia take turns writing and erasing numbers on a blackboard as follows: Bernardo starts by writing the smallest perfect square with $k+1$ digits. Every time Bernardo writes a number, Silvia erases the last k digits of it. Bernardo then writes the next perfect square, Silvia erases the last k digits of it, and this process continues until the last two numbers that remain on the board differ by at least 2. Let $f(k)$ be the smallest positive integer not written on the board. For example, if $k = 1$, then the numbers that Bernardo writes are $16, 25, 36, 49, 64$, and the numbers showing on the board after Silvia erases are $1, 2, 3, 4,$ and 6, and thus $f(1) = 5$. What is the sum of the digits of $f(2) + f(4) + f(6) + ... + f(2016)$?

$\textbf{(A)}\ 7986 \qquad \textbf{(B)}\ 8002 \qquad \textbf{(C)}\ 8030 \qquad \textbf{(D)}\ 8048 \qquad \textbf{(E)}\ 8064$

Solution

Category: AMC 12 Problems

2016 AMC 12A Answer Key

1. B
2. C
3. B
4. D
5. E
6. D
7. D
8. D
9. E
10. B
11. E
12. C
13. A
14. C
15. D
16. D
17. B
18. D
19. B
20. A
21. E
22. A
23. C
24. B
25. E

2016 AMC 12B Problems

2016 AMC 12B (Answer Key)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
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1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25

Contents

Problem 1

What is the value of $\frac{2a^{-1}+\frac{a^{-1}}{2}}{a}$ when $a=\frac{1}{2}$?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ \frac{5}{2}\qquad\textbf{(D)}\ 10\qquad\textbf{(E)}\ 20$

Solution

Problem 2

The harmonic mean of two numbers can be calculated as twice their product divided by their sum. The harmonic mean of 1 and 2016 is closest to which integer?

$\textbf{(A)}\ 2\qquad\textbf{(B)}\ 45\qquad\textbf{(C)}\ 504\qquad\textbf{(D)}\ 1008\qquad\textbf{(E)}\ 2015$

Solution

Problem 3

Let $x=-2016$. What is the value of $\Bigg\vert\Big\vert |x|-x\Big\vert-|x|\Bigg\vert-x$?

$\textbf{(A)}\ -2016\qquad\textbf{(B)}\ 0\qquad\textbf{(C)}\ 2016\qquad\textbf{(D)}\ 4032\qquad\textbf{(E)}\ 6048$

Solution

Problem 4

The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

$\textbf{(A)}\ 75\qquad\textbf{(B)}\ 90\qquad\textbf{(C)}\ 135\qquad\textbf{(D)}\ 150\qquad\textbf{(E)}\ 270$

Solution

Problem 5

The War of 1812 started with a declaration of war on Thursday, June $18, 1812$. The peace treaty to end the war was signed 919 days later, on December $24, 1814$. On what day of the week was the treaty signed?

$\textbf{(A)}\ \text{Friday}\qquad\textbf{(B)}\ \text{Saturday}\qquad\textbf{(C)}\ \text{Sunday}\qquad\textbf{(D)}\ \text{Monday}\qquad\textbf{(E)}\ \text{Tuesday}$

Solution

Problem 6

All three vertices of $\triangle ABC$ lie on the parabola defined by $y = x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length of BC?

$\textbf{(A)}\ 4\qquad\textbf{(B)}\ 6\qquad\textbf{(C)}\ 8\qquad\textbf{(D)}\ 10\qquad\textbf{(E)}\ 16$

Solution

Problem 7

Josh writes the numbers $1,2,3,\dots,99,100$. He marks out 1, skips the next number (2), marks out 3, and continues skipping and marking out the next number to the end of the list. Then he goes back to the start of his list, marks out the first remaining number (2), skips the next number (4), marks out 6, skips 8, marks out 10, and so on to the end. Josh continues in this manner until only one number remains. What is that number?

$\textbf{(A)}\ 13\qquad\textbf{(B)}\ 32\qquad\textbf{(C)}\ 56\qquad\textbf{(D)}\ 64\qquad\textbf{(E)}\ 96$

Solution

Problem 8

A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

$\textbf{(A)}\ 14.0\qquad\textbf{(B)}\ 16.0\qquad\textbf{(C)}\ 20.0\qquad\textbf{(D)}\ 33.3\qquad\textbf{(E)}\ 55.6$

Solution

Problem 9

Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

$\textbf{(A)}\ 256\qquad\textbf{(B)}\ 336\qquad\textbf{(C)}\ 384\qquad\textbf{(D)}\ 448\qquad\textbf{(E)}\ 512$

Solution

Problem 10

A quadrilateral has vertices $P(a,b)$, $Q(b,a)$, $R(-a, -b)$, and $S(-b, -a)$, where a and b are integers with $a>b>0$. The area of $PQRS$ is 16. What is $a+b$?

$\textbf{(A)}\ 4\qquad\textbf{(B)}\ 5\qquad\textbf{(C)}\ 6\qquad\textbf{(D)}\ 12\qquad\textbf{(E)}\ 13$

Solution

Problem 11

How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y=\pi x$, the line $y=-0.1$ and the line $x=5.1?$

$\textbf{(A)}\ 30\qquad\textbf{(B)}\ 41\qquad\textbf{(C)}\ 45\qquad\textbf{(D)}\ 50\qquad\textbf{(E)}\ 57$

Solution

Problem 12

All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers of consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?

$\textbf{(A)}\ 5\qquad\textbf{(B)}\ 6\qquad\textbf{(C)}\ 7\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 9$

Solution

Problem 13

Alice and Bob live 10 miles apart. One day Alice looks due north from her house and sees an airplane. At the same time Bob looks due west from his house and sees the same airplane. The angle of elevation of the airplane is 30° from Alice's position and 60° from Bob's position. Which of the following is closest to the airplane's altitude, in miles?

$\textbf{(A)}\ 3.5\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 4.5\qquad\textbf{(D)}\ 5\qquad\textbf{(E)}\ 5.5$

Solution

Problem 14

The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of $S?$

$\textbf{(A)}\ \frac{1+\sqrt{5}}{2}\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ \sqrt{5}\qquad\textbf{(D)}\ 3\qquad\textbf{(E)}\ 4$

Solution

Problem 15

All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

$\textbf{(A)}\ 312\qquad\textbf{(B)}\ 343\qquad\textbf{(C)}\ 625\qquad\textbf{(D)}\ 729\qquad\textbf{(E)}\ 1680$

Solution

Problem 16

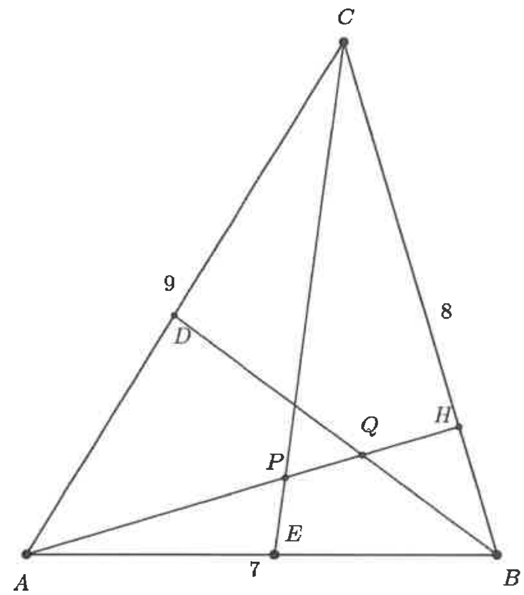
In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Solution

Problem 17

In $\triangle ABC$ shown in the figure, $AB = 7$, $BC = 8$, $CA = 9$, and $\overline{AH}$ is an altitude. Points D and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $\overline{BD}$ and $\overline{CE}$ are angle bisectors, intersecting $\overline{AH}$ at Q and P, respectively. What is PQ?



$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{5}{8}\sqrt{3} \qquad \textbf{(C)}\ \frac{4}{5}\sqrt{2} \qquad \textbf{(D)}\ \frac{8}{15}\sqrt{5} \qquad \textbf{(E)}\ \frac{6}{5}$

Solution

Problem 18

What is the area of the region enclosed by the graph of the equation $x^2+y^2=|x|+|y|?$

$\textbf{(A)}\ \pi+\sqrt{2} \qquad \textbf{(B)}\ \pi+2 \qquad \textbf{(C)}\ \pi+2\sqrt{2} \qquad \textbf{(D)}\ 2\pi+\sqrt{2} \qquad \textbf{(E)}\ 2\pi+2\sqrt{2}$

Solution

Problem 19

Tom, Dick, and Harry are playing a game. Starting at the same time, each of them flips a fair coin repeatedly until he gets his first head, at which point he stops. What is the probability that all three flip their coins the same number of times?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{7} \qquad \textbf{(C)}\ \frac{1}{6} \qquad \textbf{(D)}\ \frac{1}{4} \qquad \textbf{(E)}\ \frac{1}{3}$

Solution

Problem 20

A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B, B beat C, and C beat $A?$

$\textbf{(A)}\ 385 \qquad \textbf{(B)}\ 665 \qquad \textbf{(C)}\ 945 \qquad \textbf{(D)}\ 1140 \qquad \textbf{(E)}\ 1330$

Solution

Problem 21

Let $ABCD$ be a unit square. Let Q_1 be the midpoint of $\overline{CD}$. For $i=1,2,\dots,$ let P_i be the intersection of $\overline{AQ_i}$ and $\overline{BD}$, and let Q_{i+1} be the foot of the perpendicular from P_i to $\overline{CD}$. What is

$$\sum_{i=1}^{\infty} \text{Area of } \triangle DQ_iP_i \, ?$$

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{1}{3} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ 1$

Solution

Problem 22

For a certain positive integer n less than 1000, the decimal equivalent of $\frac{1}{n}$ is $0.\overline{abcdef}$, a repeating decimal of period of 6, and the decimal equivalent of $\frac{1}{n+6}$ is $0.\overline{wxyz}$, a repeating decimal of period 4. In which interval does n lie?

$\textbf{(A)}\ [1,200] \qquad \textbf{(B)}\ [201,400] \qquad \textbf{(C)}\ [401,600] \qquad \textbf{(D)}\ [601,800] \qquad \textbf{(E)}\ [801,999]$

Solution

Problem 23

What is the volume of the region in three-dimensional space defined by the inequalities $|x|+|y|+|z|\le1$ and $|x|+|y|+|z-1|\le1$?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{1}{3} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ 1$

Solution

Problem 24

There are exactly $77,000$ ordered quadruplets (a, b, c, d) such that $\gcd(a, b, c, d) = 77$ and $\operatorname{lcm}(a, b, c, d) = n$. What is the smallest possible value for n?

$\textbf{(A)}\ 13,860 \qquad \textbf{(B)}\ 20,790 \qquad \textbf{(C)}\ 21,560 \qquad \textbf{(D)}\ 27,720 \qquad \textbf{(E)}\ 41,580$

Solution

Problem 25

The sequence (a_n) is defined recursively by $a_0=1$, $a_1=\sqrt[19]{2}$, and $a_n=a_{n-1}a_{n-2}^2$ for $n\geq 2$. What is the smallest positive integer k such that the product $a_1a_2\cdots a_k$ is an integer?

$\textbf{(A)}\ 17 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 19 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 21$

Solution

2016 AMC 12B Answer Key

1. D
2. A
3. D
4. C
5. B
6. C
7. D
8. D
9. B
10. A
11. D
12. C
13. E
14. E
15. D
16. E
17. D
18. B
19. B
20. A
21. B
22. B
23. A
24. D
25. A

2015 AMC 12A Problems

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Instructions 1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct. 2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer. 3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will *require* the use of a calculator). 4. Figures are not necessarily drawn to scale. 5. You will have **75 minutes** working time to complete the test.
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25

Contents

Problem 1

What is the value of $(2^0 - 1 + 5^2 - 0)^{-1} \times 5$?

$\textbf{(A)}\ -125 \qquad \textbf{(B)}\ -120 \qquad \textbf{(C)}\ \frac{1}{5} \qquad \textbf{(D)}\ \frac{5}{24} \qquad \textbf{(E)}\ 25$

Solution

Problem 2

Two of the three sides of a triangle are 20 and 15. Which of the following numbers is not a possible perimeter of the triangle?

$\textbf{(A)}\ 52 \qquad \textbf{(B)}\ 57 \qquad \textbf{(C)}\ 62 \qquad \textbf{(D)}\ 67 \qquad \textbf{(E)}\ 72$

Solution

Problem 3

Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

$\textbf{(A)}\ 81 \qquad \textbf{(B)}\ 85 \qquad \textbf{(C)}\ 91 \qquad \textbf{(D)}\ 94 \qquad \textbf{(E)}\ 95$

Solution

Problem 4

The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?

$\textbf{(A)}\ \frac{5}{4} \qquad \textbf{(B)}\ \frac{3}{2} \qquad \textbf{(C)}\ \frac{9}{5} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{5}{2}$

Solution

Problem 5

Amelia needs to estimate the quantity $\frac{a}{b} - c$, where $a, b,$ and c are large positive integers. She rounds each of the integers so that the calculation will be easier to do mentally. In which of these situations will her answer necessarily be greater than the exact value of $\frac{a}{b} - c$?

$\textbf{(A)}$ She rounds all three numbers up.
$\textbf{(B)}$ She rounds a and b up, and she rounds c down.
$\textbf{(C)}$ She rounds a and c up, and she rounds b down.
$\textbf{(D)}$ She rounds a up, and she rounds b and c down.
$\textbf{(E)}$ She rounds c up, and she rounds a and b down.

Solution

Problem 6

Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2 : 1$?

$\textbf{(A)}\ 2 \qquad\textbf{(B)}\ 4 \qquad\textbf{(C)}\ 5 \qquad\textbf{(D)}\ 6 \qquad\textbf{(E)}\ 8$

Solution

Problem 7

Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

$\textbf{(A)}$ The second height is 10% less than the first.
$\textbf{(B)}$ The first height is 10% more than the second.
$\textbf{(C)}$ The second height is 21% less than the first.
$\textbf{(D)}$ The first height is 21% more than the second.
$\textbf{(E)}$ The second height is 80% of the first.

Solution

Problem 8

The ratio of the length to the width of a rectangle is $4 : 3$. If the rectangle has diagonal of length d, then the area may be expressed as kd^2 for some constant k. What is k?

$\textbf{(A)}\ \frac{2}{7} \qquad\textbf{(B)}\ \frac{3}{7} \qquad\textbf{(C)}\ \frac{12}{25} \qquad\textbf{(D)}\ \frac{16}{26} \qquad\textbf{(E)}\ \frac{3}{4}$

Solution

Problem 9

A box contains 2 red marbles, 2 green marbles, and 2 yellow marbles. Carol takes 2 marbles from the box at random; then Claudia takes 2 of the remaining marbles at random; and then Cheryl takes the last 2 marbles. What is the probability that Cheryl gets 2 marbles of the same color?

$\textbf{(A)}\ \frac{1}{10} \qquad\textbf{(B)}\ \frac{1}{6} \qquad\textbf{(C)}\ \frac{1}{5} \qquad\textbf{(D)}\ \frac{1}{3} \qquad\textbf{(E)}\ \frac{1}{2}$

Solution

Problem 10

Integers x and y with $x > y > 0$ satisfy $x + y + xy = 80$. What is x?

$\textbf{(A)}\ 8 \qquad\textbf{(B)}\ 10 \qquad\textbf{(C)}\ 15 \qquad\textbf{(D)}\ 18 \qquad\textbf{(E)}\ 26$

Solution

Problem 11

On a sheet of paper, Isabella draws a circle of radius 2, a circle of radius 3, and all possible lines simultaneously tangent to both circles. Isabella notices that she has drawn exactly $k \geq 0$ lines. How many different values of k are possible?

$\textbf{(A)}\ 2 \qquad\textbf{(B)}\ 3 \qquad\textbf{(C)}\ 4 \qquad\textbf{(D)}\ 5 \qquad\textbf{(E)}\ 6$

Solution

Problem 12

The parabolas $y = ax^2 - 2$ and $y = 4 - bx^2$ intersect the coordinate axes in exactly four points, and these four points are the vertices of a kite of area 12. What is $a + b$?

$\textbf{(A)}\ 1 \qquad\textbf{(B)}\ 1.5 \qquad\textbf{(C)}\ 2 \qquad\textbf{(D)}\ 2.5 \qquad\textbf{(E)}\ 3$

Solution

Problem 13

A league with 12 teams holds a round-robin tournament, with each team playing every other team exactly once. Games either end with one team victorious or else end in a draw. A team scores 2 points for every game it wins and 1 point for every game it draws. Which of the following is NOT a true statement about the list of 12 scores?

$\textbf{(A)}$ There must be an even number of odd scores.
$\textbf{(B)}$ There must be an even number of even scores.
$\textbf{(C)}$ There cannot be two scores of 0.
$\textbf{(D)}$ The sum of the scores must be at least 100.
$\textbf{(E)}$ The highest score must be at least 12.

Solution

Problem 14

What is the value of a for which $\frac{1}{\log_2 a} + \frac{1}{\log_3 a} + \frac{1}{\log_4 a} = 1$?

$\textbf{(A)}\ 9 \qquad\textbf{(B)}\ 12 \qquad\textbf{(C)}\ 18 \qquad\textbf{(D)}\ 24 \qquad\textbf{(E)}\ 36$

Solution

Problem 15

What is the minimum number of digits to the right of the decimal point needed to express the fraction $\frac{123456789}{2^{26}\cdot 5^4}$ as a decimal?

$\textbf{(A)}\ 4\qquad\textbf{(B)}\ 22\qquad\textbf{(C)}\ 26\qquad\textbf{(D)}\ 30\qquad\textbf{(E)}\ 104$

Solution

Problem 16

Tetrahedron $ABCD$ has $AB=5$, $AC=3$, $BC=4$, $BD=4$, $AD=3$, and $CD=\frac{12}5\sqrt2$. What is the volume of the tetrahedron?

$\textbf{(A)}\ 3\sqrt2\qquad\textbf{(B)}\ 2\sqrt5\qquad\textbf{(C)}\ \frac{24}5\qquad\textbf{(D)}\ 3\sqrt3\qquad\textbf{(E)}\ \frac{24}5\sqrt2$

Solution

Problem 17

Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

$\textbf{(A)}\ \frac{47}{256}\qquad\textbf{(B)}\ \frac{3}{16}\qquad\textbf{(C)}\ \frac{49}{256}\qquad\textbf{(D)}\ \frac{25}{128}\qquad\textbf{(E)}\ \frac{51}{256}$

Solution

Problem 18

The zeros of the function $f(x)=x^2-ax+2a$ are integers. What is the sum of the possible values of a?

$\textbf{(A)}\ 7\qquad\textbf{(B)}\ 8\qquad\textbf{(C)}\ 16\qquad\textbf{(D)}\ 17\qquad\textbf{(E)}\ 18$

Solution

Problem 19

For some positive integers p, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter p, right angles at B and C, $AB=2$, and $CD=AD$. How many different values of $p<2015$ are possible?

$\textbf{(A)}\ 30\qquad\textbf{(B)}\ 31\qquad\textbf{(C)}\ 61\qquad\textbf{(D)}\ 62\qquad\textbf{(E)}\ 63$

Solution

Problem 20

Isosceles triangles T and T' are not congruent but have the same area and the same perimeter. The sides of T have lengths of 5, 5, and 8, while those of T' have lengths of a, a, and b. Which of the following numbers is closest to b?

$\textbf{(A)}\ 3\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 5\qquad\textbf{(D)}\ 6\qquad\textbf{(E)}\ 8$

Solution

Problem 21

A circle of radius r passes through both foci of, and exactly four points on, the ellipse with equation $x^2+16y^2=16$. The set of all possible values of r is an interval $[a,b)$. What is $a+b$?

$\textbf{(A)}\ 5\sqrt2+4\qquad\textbf{(B)}\ \sqrt{17}+7\qquad\textbf{(C)}\ 6\sqrt2+3\qquad\textbf{(D)}\ \sqrt{15}+8\qquad\textbf{(E)}\ 12$

Solution

Problem 22

For each positive integer n, let $S(n)$ be the number of sequences of length n consisting solely of the letters A and B, with no more than three As in a row and no more than three Bs in a row. What is the remainder when $S(2015)$ is divided by 12?

$\textbf{(A)}\ 0\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 6\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 10$

Solution

Problem 23

Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac12$ is $\frac{a-b\pi}{c}$, where a, b, and c are positive integers and $\gcd(a,b,c)=1$. What is $a+b+c$?

$\textbf{(A)}\ 59\qquad\textbf{(B)}\ 60\qquad\textbf{(C)}\ 61\qquad\textbf{(D)}\ 62\qquad\textbf{(E)}\ 63$

Solution

Problem 24

Rational numbers a and b are chosen at random among all rational numbers in the interval $[0,2)$ that can be written as fractions $\frac nd$ where n and d are integers with $1\le d\le 5$. What is the probability that

$$(\cos(a\pi)+i\sin(b\pi))^4$$

is a real number?

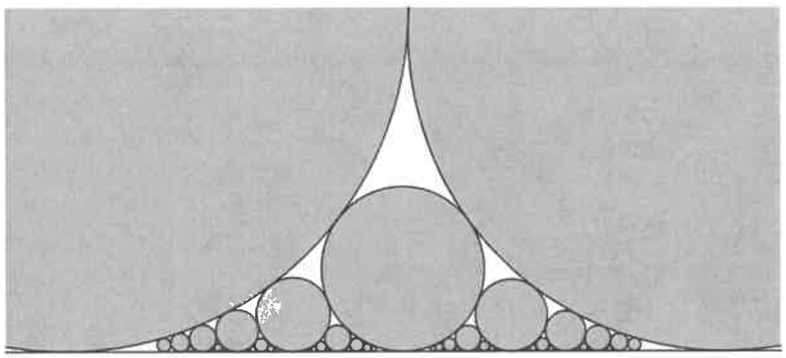
$\textbf{(A)}\ \frac{3}{50} \qquad \textbf{(B)}\ \frac{4}{25} \qquad \textbf{(C)}\ \frac{41}{200} \qquad \textbf{(D)}\ \frac{6}{25} \qquad \textbf{(E)}\ \frac{13}{50}$

Solution

Problem 25

A collection of circles in the upper half-plane, all tangent to the x-axis, is constructed in layers as follows. Layer L_0 consists of two circles of radii 70^2 and 73^2 that are externally tangent. For $k \geq 1$, the circles in $\bigcup_{j=0}^{k-1} L_j$ are ordered according to their points of tangency with the x-axis. For every pair of consecutive circles in this order, a new circle is constructed externally tangent to each of the two circles in the pair. Layer L_k consists of the 2^{k-1} circles constructed in this way. Let $S = \bigcup_{j=0}^{6} L_j$, and for every circle C denote by $r(C)$ its radius. What is

$$\sum_{C\in S} \frac{1}{\sqrt{r(C)}}?$$



$\textbf{(A)}\ \frac{286}{35} \qquad \textbf{(B)}\ \frac{583}{70} \qquad \textbf{(C)}\ \frac{715}{73} \qquad \textbf{(D)}\ \frac{143}{14} \qquad \textbf{(E)}\ \frac{1573}{146}$

Solution

See also

- AMC Problems and Solutions

2015 AMC 12A Answer Key

1. C
2. E
3. E
4. B
5. D
6. B
7. D
8. C
9. C
10. E
11. D
12. B
13. E
14. D
15. C
16. C
17. A
18. C
19. B
20. A
21. D
22. D
23. A
24. D
25. D

2015 AMC 12B Problems

2015 AMC 12B (Answer Key)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
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1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25

Contents

Problem 1

What is the value of $2-(-2)^{-2}$?

$\textbf{(A) } -2 \qquad \textbf{(B) } \frac{1}{16} \qquad \textbf{(C) } \frac{7}{4} \qquad \textbf{(D) } \frac{9}{4} \qquad \textbf{(E) } 6$

Solution

Problem 2

Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

$\textbf{(A) }\text{3:10 PM}\qquad\textbf{(B) }\text{3:30 PM}\qquad\textbf{(C) }\text{4:00 PM}\qquad\textbf{(D) }\text{4:10 PM}\qquad\textbf{(E) }\text{4:30 PM}$

Solution

Problem 3

Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

$\textbf{(A) }8\qquad\textbf{(B) }11\qquad\textbf{(C) }14\qquad\textbf{(D) }15\qquad\textbf{(E) }18$

Solution

Problem 4

David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

$\textbf{(A) } \text{David} \qquad\textbf{(B) } \text{Hikmet} \qquad\textbf{(C) } \text{Jack} \qquad\textbf{(D) } \text{Rand} \qquad\textbf{(E) } \text{Todd}$

Solution

Problem 5

The Tigers beat the Sharks 2 out of the 3 times they played. They then played N more times, and the Sharks ended up winning at least 95% of all the games played. What is the minimum possible value for N?

$\textbf{(A)}\; 35 \qquad \textbf{(B)}\; 37 \qquad \textbf{(C)}\; 39 \qquad \textbf{(D)}\; 41 \qquad \textbf{(E)}\; 43$

Solution

Problem 6

Back in 1930, Tillie had to memorize her multiplication facts from 0×0 to 12×12. The multiplication table she was given had rows and columns labeled with the factors, and the products formed the body of the table. To the nearest hundredth, what fraction of the numbers in the body of the table are odd?

$\textbf{(A)}\ 0.21 \qquad \textbf{(B)}\ 0.25 \qquad \textbf{(C)}\ 0.46 \qquad \textbf{(D)}\ 0.50 \qquad \textbf{(E)}\ 0.75$

Solution

Problem 7

A regular 15-gon has L lines of symmetry, and the smallest positive angle for which it has rotational symmetry is R degrees. What is $L+R$?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 39 \qquad \textbf{(E)}\ 54$

Solution

Problem 8

What is the value of $(625^{\log_5 2015})^{\frac{1}{4}}$?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ \sqrt[4]{2015} \qquad \textbf{(C)}\ 625 \qquad \textbf{(D)}\ 2015 \qquad \textbf{(E)}\ \sqrt[4]{5^{2015}}$

Solution

Problem 9

Larry and Julius are playing a game, taking turns throwing a ball at a bottle sitting on a ledge. Larry throws first. The winner is the first person to knock the bottle off the ledge. At each turn the probability that a player knocks the bottle off the ledge is $\tfrac{1}{2}$, independently of what has happened before. What is the probability that Larry wins the game?

$\textbf{(A)}\ \dfrac{1}{2} \qquad \textbf{(B)}\ \dfrac{3}{5} \qquad \textbf{(C)}\ \dfrac{2}{3} \qquad \textbf{(D)}\ \dfrac{3}{4} \qquad \textbf{(E)}\ \dfrac{4}{5}$

Solution

Problem 10

How many noncongruent integer-sided triangles with positive area and perimeter less than 15 are neither equilateral, isosceles, nor right triangles?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Solution

Problem 11

The line $12x+5y=60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ \dfrac{360}{17} \qquad \textbf{(C)}\ \dfrac{107}{5} \qquad \textbf{(D)}\ \dfrac{43}{2} \qquad \textbf{(E)}\ \dfrac{281}{13}$

Solution

Problem 12

Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x-a)(x-b)+(x-b)(x-c)=0$?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 15.5 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)} 16.5 \qquad \textbf{(E)}\ 17$

Solution

Problem 13

Quadrilateral $ABCD$ is inscribed in a circle with $\angle BAC=70^{\circ}, \angle ADB=40^{\circ}, AD=4,$ and $BC=6$. What is AC?

$\textbf{(A)}\ 3+\sqrt{5} \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ \dfrac{9}{2}\sqrt{2} \qquad \textbf{(D)}\ 8-\sqrt{2} \qquad \textbf{(E)}\ 7$

Solution

Problem 14

A circle of radius 2 is centered at A. An equilateral triangle with side 4 has a vertex at A. What is the difference between the area of the region that lies inside the circle but outside the triangle and the area of the region that lies inside the triangle but outside the circle?

$\textbf{(A)}\ 8-\pi \qquad \textbf{(B)}\ \pi+2 \qquad \textbf{(C)}\ 2\pi-\dfrac{\sqrt{2}}{2} \qquad \textbf{(D)}\ 4(\pi-\sqrt{3}) \qquad \textbf{(E)}\ 2\pi-\dfrac{\sqrt{3}}{2}$

Solution

Problem 15

At Rachelle's school an A counts 4 points, a B 3 points, a C 2 points, and a D 1 point. Her GPA on the four classes she is taking is computed as the total sum of points divided by 4. She is certain that she will get As in both Mathematics and Science, and at least a C in each of English and History. She thinks she has a $\tfrac{1}{6}$ chance of getting an A in English, and a $\tfrac{1}{4}$ chance of getting a B. In History, she has a $\tfrac{1}{4}$ chance of getting an A, and a $\tfrac{1}{3}$ chance of getting a B, independently of what she gets in English. What is the probability that Rachelle will get a GPA of at least 3.5?

$\textbf{(A)}\ \dfrac{11}{72} \qquad \textbf{(B)}\ \dfrac{1}{6} \qquad \textbf{(C)}\ \dfrac{3}{16} \qquad \textbf{(D)}\ \dfrac{11}{24} \qquad \textbf{(E)}\ \dfrac{1}{2}$

Solution

Problem 16

A regular hexagon with sides of length 6 has an isosceles triangle attached to each side. Each of these triangles has two sides of length 8. The isosceles triangles are folded to make a pyramid with the hexagon as the base of the pyramid. What is the volume of the pyramid?

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 162 \qquad \textbf{(C)}\ 36\sqrt{21} \qquad \textbf{(D)}\ 18\sqrt{138} \qquad \textbf{(E)}\ 54\sqrt{21}$

Solution

Problem 17

An unfair coin lands on heads with a probability of $\tfrac{1}{4}$. When tossed n times, the probability of exactly two heads is the same as the probability of exactly three heads. What is the value of n ?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 13$

Solution

Problem 18

For every composite positive integer n, define $r(n)$ to be the sum of the factors in the prime factorization of n. For example, $r(50) = 12$ because the prime factorization of 50 is 2×5^{2}, and $2 + 5 + 5 = 12$. What is the range of the function r, $\{r(n): n \text{ is a composite positive integer}\}$?

$\textbf{(A)}$ the set of positive integers
$\textbf{(B)}$ the set of composite positive integers
$\textbf{(C)}$ the set of even positive integers
$\textbf{(D)}$ the set of integers greater than 3
$\textbf{(E)}$ the set of integers greater than 4

Solution

Problem 19

In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

$\textbf{(A)}\ 12+9\sqrt{3} \qquad \textbf{(B)}\ 18+6\sqrt{3} \qquad \textbf{(C)}\ 12+12\sqrt{2} \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 32$

Solution

Problem 20

For every positive integer n, let $\text{mod}_5 (n)$ be the remainder obtained when n is divided by 5. Define a function $f: \{0,1,2,3,\dots\} \times \{0,1,2,3,4\} \to \{0,1,2,3,4\}$ recursively as follows:

$$f(i,j) = \begin{cases}\text{mod}_5 (j+1) & \text{ if } i = 0 \text{ and } 0 \le j \le 4 \text{,}\\ f(i-1,1) & \text{ if } i \ge 1 \text{ and } j = 0 \text{, and} \\ f(i-1, f(i,j-1)) & \text{ if } i \ge 1 \text{ and } 1 \le j \le 4. \end{cases}$$

What is $f(2015,2)$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Solution

Problem 21

Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose that Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 15$

Solution

Problem 22

Six chairs are evenly spaced around a circular table. One person is seated in each chair. Each person gets up and sits down in a chair that is not the same chair and is not adjacent to the chair he or she originally occupied, so that again one person is seated in each chair. In how many ways can this be done?

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 24$

Solution

Problem 23

A rectangular box measures $a \times b \times c$, where a, b, and c are integers and $1\leq a \leq b \leq c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a,b,c) are possible?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 26$

Solution

Problem 24

Four circles, no two of which are congruent, have centers at A, B, C, and D, and points P and Q lie on all four circles. The radius of circle A is $\tfrac{5}{8}$ times the radius of circle B, and the radius of circle C is $\tfrac{5}{8}$ times the radius of circle D. Furthermore, $AB = CD = 39$ and $PQ = 48$. Let R be the midpoint of $\overline{PQ}$. What is $AR+BR+CR+DR$?

$\textbf{(A)}\ 180 \qquad \textbf{(B)}\ 184 \qquad \textbf{(C)}\ 188 \qquad \textbf{(D)}\ 192 \qquad \textbf{(E)}\ 196$

Solution

Problem 25

A bee starts flying from point P_0. She flies 1 inch due east to point P_1. For $j \ge 1$, once the bee reaches point P_j, she turns 30° counterclockwise and then flies $j+1$ inches straight to point P_{j+1}. When the bee reaches P_{2015} she is exactly $a\sqrt{b}+c\sqrt{d}$ inches away from P_0, where a, b, c and d are positive integers and b and d are not divisible by the square of any prime. What is $a+b+c+d$?

$\textbf{(A)}\ 2016 \qquad \textbf{(B)}\ 2024 \qquad \textbf{(C)}\ 2032 \qquad \textbf{(D)}\ 2040 \qquad \textbf{(E)}\ 2048$

Solution

See also

2015 AMC 12B (Problems • Answer Key • Resources)	
Preceded by **2015 AMC 12A Problems**	Followed by **2016 AMC 12A Problems**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

- AMC 10
- AMC 10 Problems and Solutions
- Mathematics competition resources

2015 AMC 12B Answer Key

1. C
2. B
3. A
4. B
5. B
6. A
7. D
8. D
9. C
10. C
11. E
12. D
13. B
14. D
15. D
16. C
17. D
18. D
19. C
20. B
21. D
22. D
23. B
24. D
25. B

2015 AMC 12B Answer Key

1. C
2. B
3. A
4. B
5. B
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7. D
8. D
9. C
10. C
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12. D
13. B
14. D
15. D
16. C
17. D
18. D
19. C
20. B
21. D
22. D
23. B
24. D
25. B