## Derivative of a Function

Let $y=f(x)$ denote a function $f$. The derivative of $f$ at $x$, denoted by $f^{\prime}(x)$, read " $f$ prime of $x$," is defined by
$f^{\prime}(x)=$
provided that this limit exists. The derivative of a function $f$ gives the slope of $f$ for any value of $x$ is the domain of $f$ '.

1. Find the slope of the function $f(x)=\sqrt{x}$.

| Alternative Definition: <br> The derivative of the function $f$ at the point $x=a$ is the limit $f^{\prime}(a)=$ |  |
| :---: | :---: |

2. Use the definition $f^{\prime}(a)=\lim _{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$ to find the derivative of $f(x)=2 x+3$ at $a=-1$.

Different ways to write the derivative:
1.
2.
3.
4.

## Comparing graphs of derivative to function:


(a)
2. Sketch the graph of a function $f$ that has the following properties:
I. $\quad f(0)=0$
II. The graph of $f^{\prime}$ is shown below
III. $f$ is continuous for all $x$

3. Sketch a graph of the derivative from the function below.

## One-Sided Derivatives

A function $y=f(x)$ is differentiable on a closed interval $[a, b]$ if it has a derivative at every interior point of the interval, and if the limits

$$
\begin{array}{ll}
\lim _{h \rightarrow 0^{+}} \frac{f(a+h)-f(a)}{h} & \text { [the right-hand derivative at } a \text { ] } \\
\lim _{h \rightarrow 0^{-}} \frac{f(b+h)-f(b)}{h} & {[\text { the left-hand derivative at } b]}
\end{array}
$$

exist at the endpoints. In the right-hand derivative, $h$ is positive and $a+h$ approaches $a$ from
4. Show that the following function has left-hand and right-hand derivatives at $x=0$, but no derivative at $x=0$

$$
y= \begin{cases}x^{2}, & x \leq 0 \\ 2 x, & x>0\end{cases}
$$

