## Derivative of a Function

Let y = f(x) denote a function f. The derivative of f at x, denoted by f'(x), read "f prime of x," is defined by

f'(x) =

provided that this limit exists. The derivative of a function f gives the slope of f for any value of x is the domain of f'.

1. Find the slope of the function  $f(x) = \sqrt{x}$ .

Alternative Definition:<br/>The derivative of the<br/>function f at the point<br/>x = a is the limity<br/>f(x)f(a) =f(a) =



2. Use the definition  $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$  to find the derivative of f(x) = 2x + 3 at a = -1.

Different ways to write the derivative:

1.

4.

## Comparing graphs of derivative to function:



- 2. Sketch the graph of a function *f* that has the following properties:
  - I. f(0) = 0
  - II. The graph of f' is shown below
  - III. f is continuous for all x

3. Sketch a graph of the derivative from the function below.



## **One-Sided Derivatives**

A function y = f(x) is **differentiable on a closed interval** [a, b] if it has a derivative at every interior point of the interval, and if the limits

$$\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h} \quad \text{[the right-hand derivative at } a\text{]}$$
$$\lim_{h \to 0^-} \frac{f(b+h) - f(b)}{h} \quad \text{[the left-hand derivative at } b\text{]}$$

exist at the endpoints. In the right-hand derivative, h is positive and a + h approaches a from

4. Show that the following function has left-hand and right-hand derivatives at x = 0, but no derivative at x = 0

$$y = \begin{cases} x^2, & x \le 0\\ 2x, & x > 0 \end{cases}$$