

## Challenge Problems 3.2

The cubic polynomial  $f(x)$  passes through  $(0, 4)$  and has zeros at  $x = 1, x = 3$ , and  $x = 5$ . Find  $f(4)$ .

$$f(x) = a(x-1)(x-3)(x-5)$$

$$4 = a(0-1)(0-3)(0-5)$$

$$4 = -15a$$

$$-4/15 = a$$

$$f(x) = -\frac{4}{15}(x-1)(x-3)(x-5)$$

$$f(4) = -\frac{4}{15}(4-1)(4-3)(4-5)$$

$$= -\frac{4}{15}(3)(1)(-1)$$

$$= 12/15 = \boxed{4/5}$$

Find the remainder when dividing  $x^9 + 2x^8 + 3x^7 + 4x^6 + \dots + 8x^2 + 9x + 10$  by  $x + 2$ .

$$\begin{array}{r|rrrrrrrrrr} -2 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ & & -2 & 0 & -6 & 4 & -18 & 24 & -62 & 108 & -234 \\ \hline & 1 & 0 & 3 & -2 & 9 & -12 & 31 & -54 & 117 & -224 \end{array}$$

$$\boxed{-224}$$

Determine  $a$  and  $b$  such that 2 is a double zero (i.e.  $(x-2)^2$ ) for the polynomial:

$$x^4 + (a-2)x^3 + bx^2 + (a+b)x + 4$$

$$10a + 6b + 4 = 0$$

$$13a + 5b + 8 = 0$$

$$\begin{array}{r|rrrrr} 2 & 1 & a-2 & b & a+b & 4 \\ & & 2 & 2a & 4a+2b & 10a+6b \\ \hline & 1 & a & b+2a & 5a+3b & \underbrace{10a+6b+4}_{=0} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & a & 2a+b & 5a+3b \\ & & 2 & 2a+4 & 8a+2b+8 \\ \hline & 1 & a+2 & 4a+b+4 & \underbrace{13a+5b+8}_{=0} \end{array}$$

$$\boxed{\begin{array}{l} a = -1 \\ b = 1 \end{array}}$$

back →

$$\begin{array}{l} (10a + 6b = -4) \cdot 5 \\ (13a + 5b = -8) \cdot 6 \end{array}$$

$$-50a - 30b = 20$$

$$78a + 30b = -48$$

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$$28a = -28$$

$$a = -1$$

$$10(-1) + 6b = -4$$

$$6b = 6$$

$$b = 1$$