$\text{Divisor}\overline{)\text{Dividend}}$ with $\text{Quotient} + \frac{\text{remainder}}{\text{divisor}}$ above	Dividend = (Quotient)(Divisor) + Remainder

Warm-ups with Long Division (no calculator!)

1. $2445 \div 3 = 815$

*mult.

$$\begin{array}{r} 815 \\ 3\,\overline{)\,2445} \\ -24 \\ \hline 04 \\ -3 \\ \hline 15 \\ -15 \\ \hline 0 \end{array}$$

*subt.

*carry next digit

2. $976 \bullet 5^{-1}$

$$\frac{976}{5} = 195\frac{1}{5} = 195.2$$

$$\begin{array}{r} 195 \\ 5\,\overline{)\,976} \\ -5 \\ \hline 47 \\ -45 \\ \hline 26 \\ -25 \\ \hline 1 \end{array}$$

3. $\frac{2089}{4} = 522\frac{1}{4} = 522.25$

$$\begin{array}{r} 522 \\ 4\,\overline{)\,2089} \\ -20 \\ \hline 08 \\ -8 \\ \hline 09 \\ -8 \\ \hline 1 \end{array}$$

Use the same procedure with polynomials!

4. $(4x^3 - 9x^2 - 10x - 2) \div (x - 3)$

*mult.

focus on leading terms

$$\begin{array}{r} 4x^2 + 3x - 1 \\ x-3\,\overline{)\,4x^3 - 9x^2 - 10x - 2} \\ -(4x^3 - 12x^2) \\ \hline 3x^2 - 10x \\ -(-3x^2 - 9x) \\ \hline -x - 2 \\ -(-x + 3) \\ \hline -5 \end{array}$$

*subt

*carry next digit

$$\frac{4x^3 - 9x^2 - 10x - 2}{x - 3} = \boxed{4x^2 + 3x - 1 + \frac{-5}{x-3}}$$

Remainder Theorem:	**Factor Theorem:**
If a polynomial $f(x)$ is divided by $(x-k)$ then the remainder is $R = f(k)$. $(x-k) \rightarrow (x-3)$ find $f(3)$	A polynomial $f(x)$ has a factor $(x-k)$ if and only if $f(k) = 0$. ↑ remainder $(x-k)$ is a factor if the remainder $= 0$
Verify the Remainder Theorem for #4:	**Is (x-3) a factor of the polynomial in #4?**
$f(3) = 4(3)^3 - 9(3)^2 - 10(3) - 2$	No b/c remainder $\neq 0$

$= -5$

Remainder!

* 0 for placeholder

5. $(-x^4+5x^3-10x-4)\cdot(x+1)^{-1}$

$\frac{-x^4+5x^3-10x-4}{x+1} = \boxed{-x^3+6x^2-6x-4}$

$-x^3 \quad +6x^2 \quad -6x \quad -4$

$x+1\,\overline{)\,-x^4+5x^3+0x^2-10x-4}$

$-(-x^4-x^3)$

$6x^3+0x^2$

$-(6x^3+6x^2)$

$-6x^2-10x$

$-(-6x^2-6x)$

$-4x-4$

$-(-4x-4)$

0

Is the divisor a factor?

yes

Evaluate the polynomial for $x=-1$

$f(-1) = -(-1)^4+5(-1)^3-10(-1)-4$

$=0$

6. $\frac{(4x^3-7x^2-11x+5)}{(4x+5)}$

$x^2 \quad -3x \quad +1$

$4x+5\,\overline{)\,4x^3-7x^2-11x+5}$

$-(4x^3+5x^2)$

$-12x^2-11x$

$-(-12x^2-15x)$

$4x+5$

$-(4x+5)$

0

$=\boxed{x^2-3x+1}$

Is the divisor a factor?

yes

Evaluate the polynomial for $x=-\frac{5}{4}$

$4(-5/4)^3-7(-5/4)^2-11(-5/4)+5=0$

7. $(5x^4+2x^3-9x+12)\div(x^2-3x+4)$

$5x^2 \quad +17x \quad +31$

$x^2-3x+4\,\overline{)\,5x^4+2x^3+0x^2-9x+12}$

$-(5x^4-15x^3+20x^2)$

$17x^3-20x^2-9x$

$-(17x^3-51x^2+68x)$

$31x^2-77x+12$

$-(31x^2-93x+124)$

$16x-112$

$\boxed{=5x^2+17x+31+\frac{16x-112}{x^2-3x+4}}$

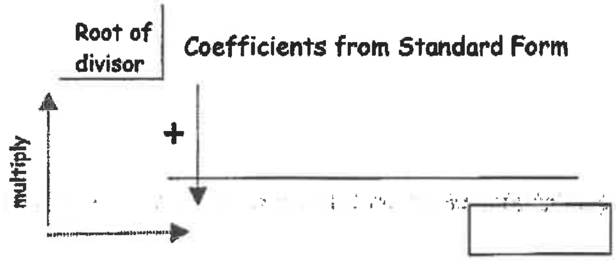
Is the divisor a factor?

No

5.2 Dividing Polynomials
Honors Algebra 2

Synthetic Division: used to divide a polynomial by a binomial divisor in the form $(x-c)$ in which c is a constant **and the coefficient of x is 1.**

* You will need a place-holder of the coefficient "zero" for each missing term!



1. $(x^3-6x^2+2x-4)\div(x-2)$

2 is root

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 2 & -4 \\ & & 2 & -8 & -12 \\ \hline & 1 & -4 & -6 & -16 \end{array}$$

$$\boxed{x^2-4x-6+\frac{-16}{x-2}}$$

Is the divisor a factor? No $(R\neq 0)$

2. $(2x^3+x^2-8x+16)\bullet(x+4)^{-1}$

-4 is root

$$\begin{array}{r|rrrr} -4 & 2 & 1 & -8 & 16 \\ & & -8 & 28 & -80 \\ \hline & 2 & -7 & 20 & -64 \end{array}$$

$$\boxed{2x^2-7x+20+\frac{-64}{x+4}}$$

Is the divisor a factor? No

3. $\dfrac{(4x^4-2x^2+x+1)}{(x-1)}$

0 placeholder

$$\begin{array}{r|rrrrr} 1 & 4 & 0 & -2 & 1 & 1 \\ & & 4 & 4 & 2 & 3 \\ \hline & 4 & 4 & 2 & 3 & 4 \end{array}$$

$$\boxed{4x^3+4x^2+2x+3+\frac{4}{x-1}}$$

Is the divisor a factor?

No

4. $(x^3-64)\div(x-4)$

$$\begin{array}{r|rrrr} 4 & 1 & 0 & 0 & -64 \\ & & 4 & 16 & 64 \\ \hline & 1 & 4 & 16 & 0 \end{array}$$

$$\boxed{x^2+4x+16}$$

Is the divisor a factor?

yes

What if the coefficient of x is not 1?

5. $\left(6x^2-5x+9\right)\div\left(2x-1\right)$

Method 1 (Rewrite)

$$\frac{6x^2-5x+9}{2x-1}$$

$$=\frac{6x^2-5x+9}{2(x-1/2)}$$

$$=\frac{3x^2-5/2x+9/2}{x-1/2}$$

$$\begin{array}{r|rrr} \frac{1}{2} & 3 & -5/2 & 9/2 \\ & & 3/2 & -1/2 \\ \hline & 3 & -1 & 4 \end{array}$$

$$3x-1+\frac{4}{x-1/2}$$

$$3x-1+\frac{8}{2x-1}$$

Method 2 (long division)

$$\begin{array}{r|l} & 3x\ \ -1 \\ \hline 2x-1 & 6x^2-5x+9 \\ & -(6x^2-3x) \\ \hline & -2x+9 \\ & -(-2x+1) \\ \hline & 8 \end{array}$$

$$3x-1+\frac{8}{2x-1}$$

Find the value of k so that the remainder for each of the following is 3.

6. $\left(x^2-x+k\right)\div\left(x-1\right)$

$$\begin{array}{r|rrr} 1 & 1 & -1 & k \\ & & 1 & 0 \\ \hline & 1 & 0 & k \end{array}$$

$$\boxed{k=3}$$

7. $\left(x^3+4x^2+x+k\right)\div\left(x+2\right)$

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 1 & k \\ & & -2 & -4 & 6 \\ \hline & 1 & 2 & -3 & k+6 \end{array}$$

$k+6=3$

$$\boxed{k=-3}$$

8. $\left(x^2+5x+7\right)\div\left(x+k\right)$

$$\begin{array}{r|rrr} -k & 1 & 5 & 7 \\ & & -k & k^2-5k \\ \hline & 1 & 5-k & k^2-5k+7 \end{array}$$

$k^2-5k+7=3$

$k^2-5k+4=0$

$(k-4)(k-1)=0$

$$\boxed{k=1,4}$$