RULE 1 Derivative of a Constant Function

If f is the function with the constant value c, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0.$$

1. Find $\frac{dy}{dx}$
   a. $f(x) = 4$

RULE 2 Power Rule for Positive Integer Powers of x

If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

   b. $f(x) = x^2 - 1$

RULE 3 The Constant Multiple Rule

If u is a differentiable function of x and c is a constant, then

$$\frac{d}{dx}(cu) = c\frac{du}{dx}.$$

RULE 4 The Sum and Difference Rule

If u and v are differentiable functions of x, then their sum and difference are differentiable at every point where u and v are differentiable. At such points,

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}.$$

c. $f(x)=\dfrac{x^3}{3}-x$

e. $f(x)=x^4-x^2+4x$

d. $y=\dfrac{x-a}{\sqrt{x}-\sqrt{a}}$

f. $y=\dfrac{w^3-w}{w}$

2. Find the values of x for which the curve has horizontal tangents.

a. $y=x^3-4x^2+x+2$

b. $f(x)=4x^3-6x^2-1$

RULE 5 The Product Rule

The product of two differentiable functions u and v is differentiable, and

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.$$

RULE 6 The Quotient Rule

At a point where $v \neq 0$, the quotient $y = u/v$ of two differentiable functions is differentiable, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

RULE 7 Power Rule for Negative Integer Powers of x

If n is a negative integer and $x \neq 0$, then

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

3. Find $\frac{dy}{dx}$

a. $y = (x^2 + 1)(x^3 + 1)$

b. $y = \frac{x^2 + 5x - 1}{x^2}$

c. $y = \dfrac{x}{x+1}$

d. $y = x(x+1)$

e. $f(x) = 3x^4(2x-1)$

f. $f(x) = \left(1 + \dfrac{1}{x^2}\right)\left(x^2 + 1\right)$

g. $f(x) = (3x-1)(2x-1)^{-1}$

4. Suppose u and v are functions of x that are differentiable at $x = 2$ and that $u(2) = 3,\ u'(2) = -3,\ v(2) = -1,$ and $v'(2) = 2.$ Find the values of the following derivatives at $x = 2$.

a. $\frac{d}{dx}(uv)$

c. $\frac{d}{dx}\left(\frac{v}{u}\right)$

b. $\frac{d}{dx}\left(\frac{u}{v}\right)$

d. $\frac{d}{dx}(3u - 2v + 2uv)$

Higher Order Derivatives

The derivative $y' = \dfrac{dy}{dx}$ is called the ______________________ of y with respect to x.

*First derivative may be differentiable

$$y'' = \frac{dy'}{dx} = \frac{d}{dx}\left(\frac{d}{dx}\right) = \frac{d^2y}{dx^2}$$

The derivative of the first derivative is called the of y with respect to x.

*If y'' ( ) is differentiable, its derivative

$$y''' = \frac{dy''}{dx} = \frac{d^3y}{dx^3}$$

The derivative of the second derivative is called the _of y with respect to x.

Prime notation continues:

$$y^{(n)} = \frac{dy^{(n-1)}}{dx} = \frac{d^ny}{dx^n}$$

to denote the of y with respect to x.

5. Find the first four derivatives of the function.

   a. $y = x^2 + x + 3$

   b. $y = \dfrac{x+1}{x}$

Instantaneous Rate of Change

6. An orange farmer currently has 200 trees yielding an average of 15 bushels of oranges per tree. She is expanding her farm at the rate of 14 trees per year, while improved husbandry is improving her average annual yield by 1.2 bushels per tree. What is the current (instantaneous) rate of increase of her total annual production of oranges?