2019-20

AB Calculus
3.4 Velocity and Other Rates of Change
3.5 Jerk

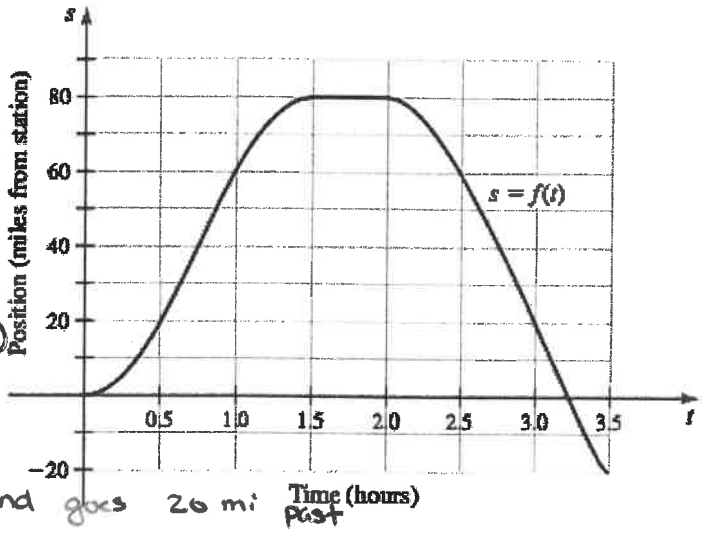
DEFINITION Instantaneous Velocity

The (instantaneous) velocity is the derivative of the position function $s = f(t)$ with respect to time. At time t the velocity is

$$v(t) = \frac{ds}{dt} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} = s'(t)$$

1. Assume a police station is located along a straight east-west freeway. At noon (t=0), a patrol car leaves the station heading east. The position function of the car $s = f(t)$ gives the location of the car in miles east ($s > 0$) or west ($s < 0$) of the station t hours after noon.

   a. Describe the location of the patrol car during the first 3.5 hours of the trip.



car moving east from 0 < t < 1.5 (noon to 1:30pm) (80 mi)

car at rest from 1.5 < t < 2 (1:30 - 2:00pm)

car moving west from 2 < t < 3.5 (2:00 - 3:30pm)

passes police station and goes 20 mi @ $t \approx 3.25$ (3:15 pm)

   b. Calculate the average velocity of the car between noon and 2:00 pm. $(0 \le t \le 2)$.

$$\text{average velocity} = \frac{s(2) - s(0)}{2 - 0} = \frac{80 - 0}{2} = 40 \text{ mph}$$

c. Calculate the displacement and average velocity of the car between 2:00 pm and 3:30 pm $(2 \le t \le 3.5)$

displacement = $s(3.5) - s(2)$ av. velo. = $\frac{s(3.5) - s(2)}{3.5 - 2}$

$= -20 - 80$

$= -100$ miles $= \frac{-100}{1.5}$

$= \frac{-200}{3}$ x

-66.667 mph

*neg → moving west (opp. direction)

d. At what time(s) is the instantaneous velocity greatest *as the car travels east?*

when greatest positive slope

↓

pos = east

around $t = 0.5$ to $t = 1$ or from 12:30 pm – 1:00 pm

e. At what time(s) is the patrol car at rest?

when inst. velocity is 0 (slope = 0)

$t = 1.5$ to $t = 2$ or 1:30 pm – 2:00 pm

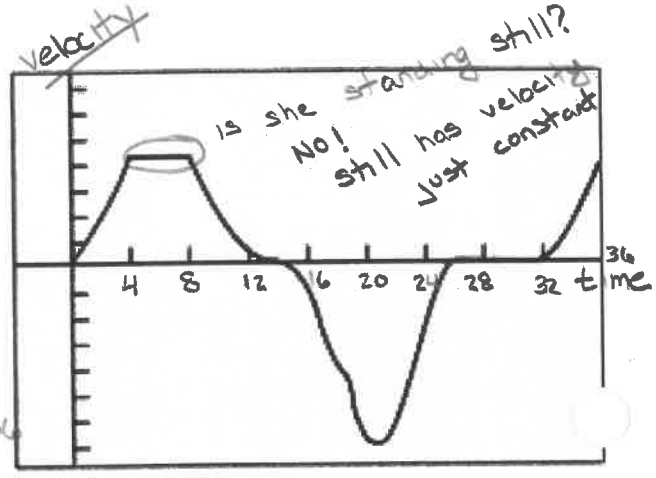
DEFINITION Speed

Speed is the absolute value of velocity.

Speed = $|v(t)|$ * don't want direction so abs. value

2. A student walks around in front of a motion detector that records her velocity at 1-second intervals for 36 seconds. She stores the data in her graphing calculator and uses it to generate the **time-velocity graph** shown below. Describe her motion as a function of time by reading the velocity graph. When is her speed a maximum?

absolute value of velocity



moves forward for first 14 sec.

constant velocity from $t = 4$ to $t = 8$

moves backward from $t = 14$ to $t = 26$

stands still from $t = 26$ to $t = 32$

moves forward from $t = 32$ to $t = 36$

speed is a maximum at $t = 20$ sec

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DEFINITION Acceleration

Acceleration is the derivative of velocity with respect to time. If a body's velocity at time t is $v(t) = ds/dt$, then the body's acceleration at time t is

$$a(t) = v'(t) = s''(t)$$

2017 AP Test #5 → No Calculator

Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

$v_P(t) = \dfrac{2t-2}{t^2-2t+10}$

(a) For $0 \le t \le 8$, when is particle P moving to the left?

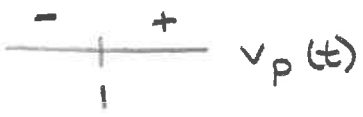
(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

a) $0 = x'_P(t) = v_P(t)$

$0 = \dfrac{2t-2}{t^2-2t+10}$

$t = 1$ $\quad t^2 - 2t + 10$ always pos

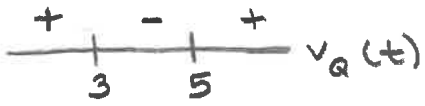


moving to the left from $[0, 1)$ b/c $v_P(t) < 0$

b) $v_Q(t) = t^2 - 8t + 15$

$0 = (t-5)(t-3)$

$t = 3$ and 5



traveling in same direction from $(1, 3) \cup (5, 8]$

b/c both $v_P(t)$ and $v_Q(t)$ are positive from $(1,3) \cup (5,8]$

c) $a_Q(t) = v_Q'(t)$

$= 2t - 8$

$a_Q(2) = -4$ $\quad v_Q(2) = 3$ $\quad$ so $a_Q(2) < 0$ and $v_Q(2) > 0$

velocity of particle Q and acceleration of particle Q have different signs @ time $t = 2$ so the particle is slowing down.

2016 AP Test #2 → Calculator Active

For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

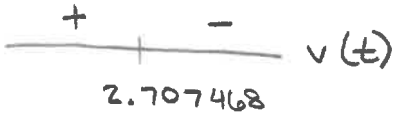
a) $v(4) = 2.978716 > 0$

$v'(4) = a(4) = -1.164 < 0$

the particle is slowing down since the velocity and acceleration of the particle have opposite signs at $t = 4$.

b) $0 = v(t)$

$t = 2.707468$



the particle changes from moving to the right then to the left @ $t = 2.707468$ b/c $v(t)$ changes sign from positive to negative at time $t = 2.707468$.