

Trig Derivative Rules:

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

1. Find $\frac{dy}{dx}$ of:

a. $y = 2 \sin x - \cos x$

$$\begin{aligned} y' &= 2 \cos x - (-\sin x) \\ &= 2 \cos x + \sin x \end{aligned}$$

c. $y = \frac{\cos x}{1 + \sin x}$

$$\begin{aligned} y' &= \frac{(1 + \sin x)(-\sin x) - (\cos x)(\cos)}{(1 + \sin x)^2} \\ &= \frac{-\sin x - \sin^2 x - \cos^2 x}{1 + 2\sin x + \sin^2 x} \\ &= \frac{-(\sin x + \sin^2 x + \cos^2 x)}{1 + 2\sin x + \sin^2 x} \end{aligned}$$

b. $y = x \sin x$

$$\begin{aligned} y' &= x(\cos x) + \sin x(1) \\ &= x \cos x + \sin x \end{aligned}$$

$$= \frac{-(\sin x + 1)}{(1 + \sin x)^2}$$

$$= \frac{-1}{1 + \sin x}$$

AB Calculus

3.5 Derivative of Trigonometric Functions

Derivatives of Other Basic Trig Functions

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

2. Find $\frac{dy}{dx}$ of:

a. $y = 3x + x \tan x$

c. $y = \sin x \cos x$

$$y' = 3 + [x(\sec^2 x) + \tan x(1)] \quad y' = \sin x(-\sin x) + \cos x(\cos x)$$

$$= 3 + x \sec^2 x + \tan x$$

$$= -\sin^2 x + \cos^2 x$$

b. $y = \frac{1-\sin x}{1+\sin x}$

d. $y = \sec x + \csc x$

$$y' = \frac{(1+\sin x)(-\cos x) - (1-\sin x)(\cos x)}{(1+\sin x)^2}$$

$$= \frac{-\cos x - \cancel{\cos x \sin x} - \cos x + \cancel{\cos x \sin x}}{(1+\sin x)^2}$$

$$= \frac{-2\cos x}{(1+\sin x)^2}$$

$$y' = \sec x \tan x + (-\csc x \cot x)$$

$$= \sec x \tan x - \csc x \cot x$$

$$\text{e. } y = \frac{\cot x}{4}$$

$$y = \frac{1}{4} \cot x$$

* remember coefficients

make as easy for self as possible
↳ don't need quotient rule

$$y' = -\frac{1}{4} \csc^2 x$$

3. Find the first 4 higher order derivatives of $y = \cos x$

$$y = \cos x$$

$$y' = -\sin x$$

$$y'' = -\cos x$$

$$y''' = \sin x$$

$$y^{(4)} = \cos x$$

}

Helpful at
end of year!

4. Find equations for the lines that are tangent and normal to the graph of

$$f(x) = \frac{\tan x}{x} \text{ at } x=2.$$

↓
perpendicular

$m = \text{neg reciprocal}$

$$f'(x) = \frac{x(\sec^2 x) - \tan x(1)}{x^2}$$

$$= \frac{x \sec^2 x - \tan x}{x^2}$$

$$f(2) = \frac{\tan(2)}{2} \approx -1.093$$

(2, -1.093)

$$f'(2) = \frac{2 \sec^2(2) - \tan(2)}{2^2}$$

$$\approx 3.433$$

slope of normal line:

$$m = -\frac{1}{3.433} \approx -0.291$$

normal line

$$y - (-1.093) = -0.291(x - 2)$$

$$y = -0.291x + 1.675$$

tangent line:

$$y - (-1.093) = 3.433(x - 2)$$

$$y = 3.433x - 5.773$$