

3.5 Perform Basic Matrix Operations

A matrix is a rectangular arrangement of numbers in rows and columns.

- ❖ The dimensions of a matrix with m rows and n columns are $m \times n$.
- ❖ The numbers in a matrix are called elements.

A matrix can be named with a capital letter and its dimensions. For instance, $A_{3 \times 2}$.

Two matrices are said to be equal when they have the same dimensions and the elements in the corresponding positions are the same.

Find each of the missing variables.

<p>1. $\begin{bmatrix} 2 & 3 \\ m & a \end{bmatrix} = \begin{bmatrix} t & h \\ -4 & 34 \end{bmatrix}$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\begin{aligned} m &= -4 \\ a &= 34 \\ t &= 2 \\ h &= 3 \end{aligned}$ </div>	<p>2. $\begin{bmatrix} 5x & \\ 5x+4y & \end{bmatrix} = \begin{bmatrix} y & \\ 10 & \end{bmatrix}$</p> $\begin{aligned} 5x &= y & 5x+4y &= 10 \\ & & y+4y &= 10 \\ & & 5y &= 10 \\ & & y &= 2 \end{aligned}$ <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $x = 2/5 \quad y = 2$ </div>	<p>3. $\begin{bmatrix} x^2 & 7 & 9 \\ 5 & 12 & 6 \end{bmatrix} = \begin{bmatrix} 25 & 7 & y \\ 5 & 2z & 6 \end{bmatrix}$</p> $\begin{aligned} x^2 &= 25 & 2z &= 12 \\ x &= \pm 5 & z &= 6 \end{aligned}$ <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\begin{aligned} x &= \pm 5 \\ y &= 9 \\ z &= 6 \end{aligned}$ </div>
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KEY CONCEPT *For Your Notebook*

Adding and Subtracting Matrices

To add or subtract two matrices, simply add or subtract elements in corresponding positions. You can add or subtract matrices only if they have the same dimensions.

Adding Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

Subtracting Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$

Perform the indicated operations.

<p>4. $\begin{bmatrix} 2 & -1 \\ 3 & 7 \\ 14 & 9 \end{bmatrix} + \begin{bmatrix} -6 & 9 \\ 7 & -11 \\ -8 & 17 \end{bmatrix}$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\begin{bmatrix} -4 & 8 \\ 10 & -4 \\ 6 & 26 \end{bmatrix}$ </div>	<p>5. $\begin{bmatrix} -7 & x & -7 \\ 24 & 9 & 5 \\ 2 & 4 & 0 \end{bmatrix} + \begin{bmatrix} -3 & -x^2 & -4 \\ +76 & -3 & -x \\ -18 & -9 & +7 \end{bmatrix}$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\begin{bmatrix} -10 & x-x^2 & -11 \\ 100 & 6 & 5-x \\ -16 & -5 & 7 \end{bmatrix}$ </div>	<p>6. $\begin{bmatrix} 3 & -5 \end{bmatrix} + \begin{bmatrix} 6 \\ -8 \end{bmatrix}$</p> <p>$1 \times 2 + 2 \times 1$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> <p style="text-align: center;">Not Possible!</p> </div>
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Scalar multiplication: a matrix can be multiplied by a constant (called a scalar). Every element is multiplied by the scalar and a new matrix is formed.

<p>7. $A = -3 \begin{bmatrix} 2 & -16 & 1 \\ -9 & 5 & 0 \\ 4 & 22 & -7 \end{bmatrix}$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\begin{bmatrix} -6 & 48 & -3 \\ 27 & -15 & 0 \\ -12 & -66 & 21 \end{bmatrix}$ </div> <p>* What is element $A_{3,2}$? -66</p>	<p>8. $B = \frac{2}{3} \begin{bmatrix} 2 & -9 \\ 5 & -1 \\ 8 & -4 \end{bmatrix}$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\begin{bmatrix} \frac{4}{15} & -6 \\ \frac{16}{3} & -\frac{1}{6} \end{bmatrix}$ </div> <p>* What is element $B_{1,2}$? -6</p>
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Find each of the missing variables.

<p>9. $4 \begin{bmatrix} x & y-1 \\ 3 & z \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 6z & x+y \end{bmatrix}$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\begin{bmatrix} 4x & 4y-4 \\ 12 & 4z \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 6z & x+y \end{bmatrix}$ </div> <p> $4x = 20$ $4y - 4 = 8$ $4z = 5 + 3$ $x = 5$ $y = 3$ $z = 2$ </p>	<p>10. $\begin{bmatrix} x \\ 7z \\ 2y \end{bmatrix} - \begin{bmatrix} 4z \\ -3y \\ 3x \end{bmatrix} + \begin{bmatrix} -2y \\ 2x \\ -5z \end{bmatrix} = \begin{bmatrix} -4 \\ 11 \\ 18 \end{bmatrix}$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\begin{bmatrix} x - 4z - 2y \\ 7z + 3y + 2x \\ 2y - 3x - 5z \end{bmatrix} = \begin{bmatrix} -4 \\ 11 \\ 18 \end{bmatrix}$ </div> <p> $x - 2y - 4z = -4$ $2x + 3y + 7z = 11$ $-3x + 2y - 5z = 18$ </p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $(2, 7, -2)$ </div>
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