Direct Variation

y **varies directly** as x or y is **directly proportional** to x, if there exists a nonzero real number k, called the **constant of variation,** such that for all x,

$$y = kx$$

Inverse Variation as nth Power

Let n be a positive real number. Then y **varies inversely as the n the power** of x or y is **inversely proportional to the nth power** of x, if for all x there exists a nonzero real number k such that

$$y = \frac{k}{x^n}$$

Joint Variation

Let m and n be real numbers. Then y **varies jointly** as the nth power of x and the mth power of z if for all x and z, there exists a nonzero real number k such that

$$y = kx^n z^m$$

I. The area of a rectangle varies directly as its length. If the area is 50 m^2 when the length is 10 m, find the area when the length is 25 m.

$A = kl$ $\quad$ $A = 5l$ $\quad$ $A = 5(25)$

$50 = k(10)$ $\quad$ $= \boxed{125\ m^2}$

$5 = k$

II. In a certain manufacturing process, the cost of producing a single item varies inversely as the square of the number of items produced. If 100 items are produced, each costs $2. Find the cost per item if 400 itesm are produced. C = cost $\quad$ n = # items produced

$C = \frac{k}{n^2}$ $\quad$ $C = \frac{20{,}000}{n^2}$ $\quad$ $C = \frac{20{,}000}{400^2}$

$2 = \frac{k}{100^2}$ $\quad$ $C = \$\,1/8$

$20{,}000 = k$ $\quad$ $\boxed{= \$\,0.125}$

III. At a given speed, the distance traveled by a vehicle varies directly as the time. If a vehicle travels 156 mi in 3 hr, find the distance it will travel in 5 hr at the same average speed.

$d = kt$ $\quad$ $d = 52t$ $\quad$ $d = 52(5)$

$156 = k(3)$ $\quad$ $= \boxed{260\ mi}$

$52 = k$

IV. The volume of a cylinder varies directly as the height and the square of the radius. A cylinder with radius 5 cm and height 10 cm has volume 785 cm^3. Find the volume of a cylinder with radius 10 cm and height 15 cm.

$V = khr^2$

$785 = k(10)(5)^2$

$\frac{157}{50} = k$

$V = \frac{157}{50} hr^2$

$V = \frac{157}{50}(15)(10)^2$

$V = \boxed{4710 \text{ cm}^3}$

V. In a certain manufacturing process, the cost of producing a single item varies inversely as the square of the number of items produced. If 100 items are produced, each costs \$1.50. Find the cost per item if 250 items are produced.

$C = \frac{k}{n^2}$

$1.50 = \frac{k}{100^2}$

$15000 = k$

$C = \frac{15000}{n^2}$

$C = \frac{15000}{250^2}$

$\boxed{C = \$0.24}$

VI. Vanessa has determined that the number of children enrolled in her day care center varies directly with the amount she spends on advertising per year and inversely with her weekly charge for each child. This year she spent \$900 on advertising, charged \$150 per child per week, and had 51 children enrolled. Next year, if she spends \$1000 on advertising and charges \$170 per week, how many children should she expect to enroll?

n = # of children enrolled

a = advertising

c = weekly charge

$n = \frac{ka}{c}$

$51 = \frac{900k}{150}$

$17/2 = k$

$n = \frac{17/2\, a}{c}$

$n = \frac{17/2\,(1000)}{170}$

$\boxed{n = 50}$

extension: Is it more profitable for Vanessa to charge \$150 and spend \$900 on advertising or charge \$170 and spend \$1000?

$P_{150} = 51(150) - 900 = 6750$

$P_{170} = 50(170) - 1000 = 7500$

charge \$170 and spend \$1000