Algebra 2 Notes **Name** _____

3.8 Use Inverse Matrices to Solve Linear Systems

The $n \times n$ Identity Matrix is a matrix with 1's on the main diagonal and 0's elsewhere.

2×2 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3×3 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $AI = A$ and $AA^{-1} = I$.

Ex. Find the product: $\begin{bmatrix} 5 & -2 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -7 & 4 \end{bmatrix}$

$2 \times 2 \quad 2 \times 2$

$$= \begin{bmatrix} 5+0 & 0-2 \\ -7+0 & 0+4 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -7 & 4 \end{bmatrix}$$

Two matrices A and B are inverses of each other if their product (in both orders) is the $n \times n$ identity matrix. In other words, $AB = I$ and $BA = I$.

- An $n \times n$ matrix has an inverse if and only if its determinant $\neq 0$
- The symbol for the inverse of A is A^{-1}.

KEY CONCEPT *For Your Notebook*

The Inverse of a 2 × 2 Matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad - cb \neq 0.$$

Ex. Find the inverse of the matrix: $\begin{bmatrix} 3 & 8 \\ 2 & 5 \end{bmatrix}$

$$A^{-1} = \frac{1}{3(5) - 8(2)}\begin{bmatrix} 5 & -8 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{15 - 16}\begin{bmatrix} 5 & -8 \\ -2 & 3 \end{bmatrix}$$

$$= -1\begin{bmatrix} 5 & -8 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -5 & 8 \\ 2 & -3 \end{bmatrix}$$

Find the inverse of the matrix, if it exists.

1. $B = \begin{bmatrix} 3 & 2 \\ 4 & 2 \end{bmatrix}$

$$B^{-1} = \frac{1}{3(2)-4(2)} \begin{bmatrix} 2 & -2 \\ -4 & 3 \end{bmatrix}$$

$$= \frac{1}{6-8} \begin{bmatrix} 2 & -2 \\ -4 & 3 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 2 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & -3/2 \end{bmatrix}$$

2. $\begin{bmatrix} 7 & 14 \\ 3 & 6 \end{bmatrix}$ $\det = 7(6) - 3(14)$

$= 42 - 42$

$= 0$

Inverse does not exist

Solve the matrix equation.

- Begin by finding the inverse of the coefficient matrix.
- To solve the equation, multiply both sides of the equation by A^{-1} ***on the left.***

★ $AA^{-1} = I$ ★

$Ax = B$

$A^{-1}Ax = A^{-1}B$

$x = A^{-1}B$

3. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 5 & 1 \\ 2 & 1 \end{bmatrix}$

$Ax = B$

inverse:

$\det = 4 - 3$

$= 1$

$A^{-1} = 1 \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$A^{-1}Ax = A^{-1}B$

$Ix = A^{-1}B$

$x = A^{-1}B$

$x = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 2 & 1 \end{bmatrix}$

$x = \begin{bmatrix} 10-2 & 2-1 \\ -15+4 & -3+2 \end{bmatrix}$

$x = \begin{bmatrix} 8 & 1 \\ -11 & -1 \end{bmatrix}$

Use an inverse matrix to solve the linear system.

- Write the system as a matrix equation $AX = B$.
- Find the inverse of matrix A.
- Multiply each side of the equation by A^{-1} **on the left** to find the solution:

4. $-2x + 3y = -11$

$5x + y = 19$

$$\underset{A}{\begin{bmatrix} -2 & 3 \\ 5 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} -11 \\ 19 \end{bmatrix}}$$

$\det A = -2 - 15 = -17$

$$A^{-1} = -\frac{1}{17} \begin{bmatrix} 1 & -3 \\ -5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1/17 & 3/17 \\ 5/17 & 2/17 \end{bmatrix}$$

$Ax = B$

$x = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/17 & 3/17 \\ 5/17 & 2/17 \end{bmatrix} \begin{bmatrix} -11 \\ 19 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11/17 + 57/17 \\ -55/17 + 38/17 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 68/17 \\ -17/17 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$(4, -1)$

Calculator

TI nspire cx CAS

select button w/ diagram $|\square|$ $\begin{Bmatrix}\square\\\square\end{Bmatrix}$ * next to the "9" *

select matrix w/ proper dimensions

TI 84

"2nd" and "x^{-1}" * in blue letters above the button it says "matrix" *

Edit

Enter on "A"

Type in dimensions

Arrow down and plug in elements

Repeat process to enter a second matrix under "B"

To perform operations:

"2nd" and "x^{-1}"

Enter on matrix name you would like

select operation

"2nd" and "x^{-1}"

Enter on matrix name you would like