

## 3.8 Use Inverse Matrices to Solve Linear Systems

The  $n \times n$  \_\_\_\_\_ is a matrix with 1's on the main diagonal and 0's elsewhere.

2×2 Identity Matrix

$$I = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$$

3×3 Identity Matrix

$$I = \begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$$

If  $A$  is any  $n \times n$  matrix and  $I$  is the  $n \times n$  identity matrix, then \_\_\_\_\_ and \_\_\_\_\_.

Ex. Find the product:  $\begin{bmatrix} 5 & -2 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Two matrices  $A$  and  $B$  are \_\_\_\_\_ of each other if their product (in both orders) is the  $n \times n$  identity matrix. In other words,  $AB = I$  and  $BA = I$ .

- An  $n \times n$  matrix has an inverse if and only if \_\_\_\_\_.
- The symbol for the inverse of  $A$  is \_\_\_\_\_.

**KEY CONCEPT***For Your Notebook***The Inverse of a 2 × 2 Matrix**

The inverse of the matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad - cb \neq 0.$$

Ex. Find the inverse of the matrix:  $\begin{bmatrix} 3 & 8 \\ 2 & 5 \end{bmatrix}$

$$A^{-1} = \text{_____} \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$$

Find the inverse of the matrix, if it exists.

1.  $\begin{bmatrix} 3 & 2 \\ 4 & 2 \end{bmatrix}$

2.  $\begin{bmatrix} 7 & 14 \\ 3 & 6 \end{bmatrix}$

Solve the matrix equation.

➤ Begin by finding the inverse of the coefficient matrix.

➤ To solve the equation, multiply both sides of the equation by  $A^{-1}$  on the left.

3.  $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 5 & 1 \\ 2 & 1 \end{bmatrix}$

Use an inverse matrix to solve the linear system.

➤ Write the system as a matrix equation  $AX = B$ .

➤ Find the inverse of matrix  $A$ .

➤ Multiply each side of the equation by  $A^{-1}$  on the left to find the solution:

4. 
$$\begin{aligned} -2x + 3y &= -11 \\ 5x + y &= 19 \end{aligned}$$