One-to-One Functions

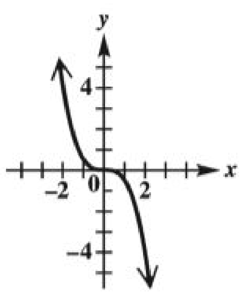
In a one-to one function, each x-value corresponds to only ___________ y-value, and each y-value corresponds to only ____________ x-value.

1. Determine whether each function is one-to-one.
   a. $f(x) = -3x + 7$
   b. $f(x) = \sqrt{49 - x^2}$

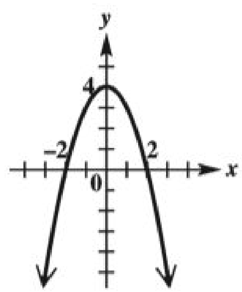
Horizontal Line Test

2. Determine whether each graph is the graph of a one-to-one function.

(a)



(b)



Inverse Functions

Inverse Function
Let f be a one-to-one function. Then g is the **inverse function** of f if

$(f \circ g)(x) = x$ for every x in the domain of g,

and $(g \circ f)(x) = x$ for every x in the domain of f.

The condition that f is one-to-one in the definition of inverse function is essential. Otherwise, g will not define a ______________.

3. Let functions f and g be defined respectively by

$$f(x) = 2x + 5 \text{ and } g(x) = \tfrac{1}{2}x - 5$$

Is g the inverse function of f?

***By the definition of inverse function, the ______________ of f is the ______________ of f^{-1}, and the ______________ of f is the ____________ of f^{-1}**

4. Find the inverse of each function that is one-to-one.

(a) $F = \{(-2,-8), (-1,-1), (0,0), (1,1), (2,8)\}$

(b) $G = \{(-2,5), (-1,2), (0,1), (1,2), (2,5)\}$

Equations of Inverses

The inverse of a one-to-one function is found by interchanging the x- and y-values of each of its ordered pairs. The equation of the inverse function defined by $y = f(x)$ is found in the same way.

5. Determine whether each equation defines a one-to-one function. If so, find the equation of the inverse.
   a. $f(x) = |x|$
   b. $y = 4x - 7$
   c. $h(x) = x^3 + 2$

6. The following rational function is one-to-one. Find its inverse.

$$f(x) = \frac{-3x+1}{x-5},\ x \neq 5$$

7. Let $f(x) = x^2 + 4,\ x \leq 0$. Find $f^{-1}(x)$.

Important Facts about Inverses:

1. If f is one-to-one, then f^{-1} ______________
2. The domain of f is the ______________ of f^{-1}, and the range of f is the ______________ of f^{-1}
3. If the point (a, b) lies on the graph of f, then the point ______________ lies on the graph of f^{-1}. The graphs of f and f^{-1} are reflections of each other across the line ______________.
4. To find the equation for f^{-1}, replace $f(x)$ with y, interchange ________ and ________, and solve for y. This gives $f^{-1}(x)$.