4.1 Inverse Functions

■ **One-to-One Functions** ■ **Inverse Functions** ■ **Equations of Inverses**

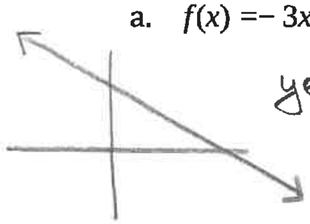
One-to-One Functions

In a one-to one function, each x-value corresponds to only one y-value, and each

y-value corresponds to only one x-value.

1. Determine whether each function is one-to-one.

   a. $f(x) = -3x + 7$

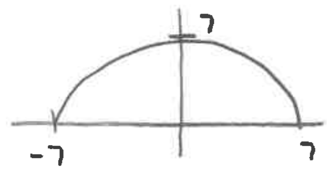


yes one to one

   b. $f(x) = \sqrt{49 - x^2}$

$y^2 = 49 - x^2 \Rightarrow y^2 + x^2 = 49$

* circle



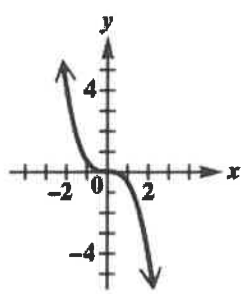
Not one-to-one

Horizontal Line Test

A function is one-to-one if every horizontal line intersects the graph of the function at most once.

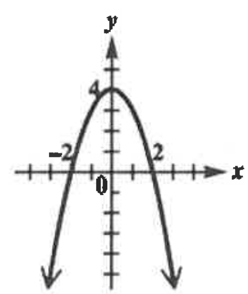
2. Determine whether each graph is the graph of a one-to-one function.

(a)



one-to-one

(b)



not one-to-one

*In general, a function that is either increasing or decreasing on its entire domain, such as $f(x) = -x$, $g(x) = x^3$, $h(x) = \frac{1}{x}$, must be one-to-one.

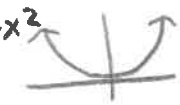
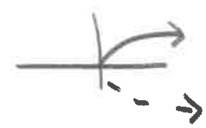
Inverse Functions

Inverse Function
Let f be a one-to-one function. Then g is the **inverse function** of f if

$(f \circ g)(x) = x$ for every x in the domain of g,

and $(g \circ f)(x) = x$ for every x in the domain of f.

The condition that f is one-to-one in the definition of inverse function is essential. Otherwise, g will not define a function

ex) $y = x^2$  $y^{-1} = \sqrt{x}$ 

3. Let functions f and g be defined respectively by
$$f(x) = 2x + 5 \text{ and } g(x) = \tfrac{1}{2}x - 5$$
Is g the inverse function of f?

$f(g(x)) = 2(\tfrac{1}{2}x - 5) + 5$
$= x - 10 + 5$
$= x - 5$

$\boxed{\text{Not inverses}}$

***By the definition of inverse function, the** domain **of** f **is the** range **of** f^{-1}**, and the** range **of** f **is the** domain **of** f^{-1}

★ swap points

f (a, b)
f^{-1} (b, a)

4. Find the inverse of each function that is one-to-one.

(a) $F = \{(-2,-8), (-1,-1), (0,0), (1,1), (2,8)\}$

$F^{-1} = \{(-8,-2), (-1,-1), (0,0), (1,1), (8,2)\}$

(b) $G = \{(-2,5), (-1,2), (0,1), (1,2), (2,5)\}$

Not one-to-one

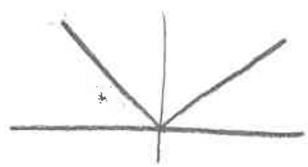
Equations of Inverses

The inverse of a one-to-one function is found by interchanging the x- and y-values of each of its ordered pairs. The equation of the inverse function defined by $y = f(x)$ is found in the same way.

5. Determine whether each equation defines a one-to-one function. If so, find the equation of the inverse.

a. $f(x) = |x|$

not one-to-one



b. $y = 4x - 7$

$x = 4y - 7$

$x + 7 = 4y$

$\frac{x+7}{4} = y$

$y^{-1} = \frac{1}{4}x + \frac{7}{4}$

c. $h(x) = x^3 + 2$

$x = y^3 + 2$

$x - 2 = y^3$

$\sqrt[3]{x-2} = y$

$h^{-1}(x) = \sqrt[3]{x-2}$

6. The following rational function is one-to-one. Find its inverse.

$f(x) = \frac{-3x+1}{x-5},\ x \neq 5$ Know y in inverse *

$x = \frac{-3y+1}{y-5}$

$x(y-5) = -3y + 1$

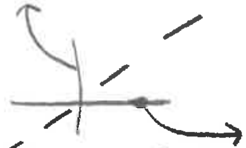
$xy - 5x = -3y + 1$

$xy + 3y = 5x + 1$

$y(x+3) = 5x + 1$

$y = \frac{5x+1}{x+3}$

$f^{-1}(x) = \frac{5x+1}{x+3},\quad x \neq -3 \quad y \neq 5$



7. Let $f(x) = x^2 + 4,\ x \le 0$. Find $f^{-1}(x)$.

$x = y^2 + 4$ * neg b/c restricting so $x \le 0$

$x - 4 = y^2$ $-\sqrt{x-4} = f^{-1}(x)$

$\pm\sqrt{x-4} = y$

Important Facts about Inverses:

1. If f is one-to-one, then f^{-1} __exists__

2. The domain of f is the __range__ of f^{-1}, and the range of f is the __domain__ of f^{-1}

3. If the point (a, b) lies on the graph of f, then the point __(b, a)__ lies on the graph of f^{-1}. The graphs of f and f^{-1} are reflections of each other across the line __$y = x$__.

4. To find the equation for f^{-1}, replace $f(x)$ with y, interchange __x__ and __y__, and solve for y. This gives $f^{-1}(x)$.