

AB Calculus
4.2 Implicit Differentiation

Recall:

- Determine the derivative of $y = x^2$

$$\frac{dy}{dx} = 2x \quad \overbrace{\frac{dx}{dx}}^1 \quad \frac{dy}{dx} = 2x$$

Implicitly Defined Functions:

ex: $x^3 + y^3 - 9xy = 0$ * can't solve for y

- Find $\frac{dy}{dx}$ of the following:

a. $y^2 = x$

$$2y \frac{dy}{dx} = 1 \frac{dx}{dx}$$

$$2y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

b. $x^3 + y^2 = 8xy$

check: * can solve for y

$$y = \sqrt{x}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

* same b/c $y^2 = x$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{y^2}} = \frac{1}{2y}$$

$$3x^2 + 2y \frac{dy}{dx} = 8[1(y) + x(1) \frac{dy}{dx}]$$

$$3x^2 + 2y \frac{dy}{dx} = 8y + 8x \frac{dy}{dx}$$

$$3x^2 - 8y = 8x \frac{dy}{dx} - 2y \frac{dy}{dx}$$

$$3x^2 - 8y = \frac{dy}{dx}(8x - 2y)$$

$$\frac{3x^2 - 8y}{8x - 2y} = \frac{dy}{dx}$$

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c. $x = \sin y$ * chain rule

$$1 = \cos y (1) \frac{dy}{dx}$$

$$\frac{1}{\cos y} = \frac{dy}{dx}$$

e. $\frac{x}{y} + \pi = x$ * quotient

$$\frac{y(1) - x(1) \frac{dy}{dx}}{y^2} = 1$$

$$y - x \frac{dy}{dx} = y^2$$

$$\frac{y - y^2}{x} = \frac{dy}{dx}$$

d. $x + \sin y = xy$

$$1 + \cos y (1) \frac{dy}{dx} = (1)y + x(1) \frac{dy}{dx}$$

$$1 + \cos y \frac{dy}{dx} = y + x \frac{dy}{dx}$$

$$1 - y = x \frac{dy}{dx} - \cos y \frac{dy}{dx}$$

$$1 - y = \frac{dy}{dx} (x - \cos y)$$

$$\frac{1 - y}{x - \cos y} = \frac{dy}{dx}$$

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Challenge: Find $\frac{dy}{dx}$

f. $x^2 = \frac{x-y}{x+y}$

$$2x = \frac{(x+y)(1 - \frac{dy}{dx}) - (x-y)(1 + \frac{dy}{dx})}{(x+y)^2}$$

$$2x = \frac{x+y - \frac{dy}{dx}(x+y) - x+y + \frac{dy}{dx}(x-y)}{(x+y)^2}$$

$$2x(x+y)^2 = 2y - \frac{dy}{dx}(x+y) - \frac{dy}{dx}(x-y)$$

$$2y - 2x(x+y)^2 = \frac{dy}{dx}(x+y) + \frac{dy}{dx}(x-y)$$

$$2y - 2x(x+y)^2 = \frac{dy}{dx}(x+y + x - y)$$

$$\frac{2y - 2x(x+y)^2}{2x} = \frac{dy}{dx}$$

$$\frac{y - x(x+y)^2}{x} = \frac{dy}{dx}$$

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Steps for Implicit Differentiation:

1. Differentiate both sides of equation w/ respect to x
2. Simplify and get $\frac{dy}{dx}$ on one side
3. Factor $\frac{dy}{dx}$
4. Solve for $\frac{dy}{dx}$

2. Find the slope of the circle $x^2 + y^2 = 25$ at the point $(3, -4)$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2(3) + 2(-4) \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-6}{-8} = \frac{3}{4}$$

* plug in point as soon as possible

3. Find the tangent and normal to the ellipse $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$

$$2x - [(1)y + x \frac{dy}{dx}] + 2y \frac{dy}{dx} = 0$$

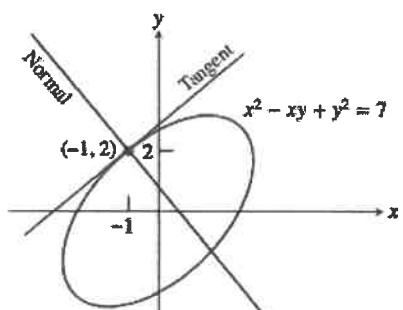
$$2x - y - x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$2(-1) - (2) - (-1) \frac{dy}{dx} + 2(2) \frac{dy}{dx} = 0$$

$$-2 - 2 + \frac{dy}{dx} + 4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{4}{5} \quad \text{slope} = \frac{4}{5}$$

$$y = 2 + \frac{4}{5}(x+1) \text{ tangent}$$



$$\text{slope normal} = -\frac{5}{4}$$

$$y = 2 - \frac{5}{4}(x+1) \text{ normal}$$

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Challenge Problems!

1. If $\frac{dy}{dx} = 1 + \sin y$, then $\frac{d^2y}{dx^2} =$

$$\frac{dy}{dx^2} = \cos y \frac{dy}{dx} \quad * \text{ know } \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \cos y (1 + \sin y)$$

2. Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 8$

$$6x^2 - 6y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-6x^2}{-6y} = \frac{x^2}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(2x) - x^2(1) \frac{dy}{dx}}{y^2} \quad * \text{ know } \frac{dy}{dx}$$

$$= \frac{2xy - x^2(x^2/y)}{y^2} \rightarrow : \frac{2xy^2 - x^4}{y} \cdot \frac{1}{y^2}$$

$$= \frac{2xy - x^4/y}{y^2}$$

$$= \frac{2xy^2 - x^4}{y^3}$$

$$= \frac{2xy^2 - x^4}{y^3}$$

