

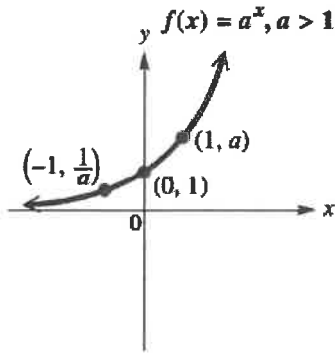
4.2 Exponential Functions
Honors Algebra 2 with Trig

1. Approximate the following with a calculator. Round answers to the nearest thousandth.

a. $5^{-1.5} \approx 0.089$

b. $e^{2.75} \approx 15.643$

Parent Graph of an exponential:



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal Asymptote: $y = 0$

Key points: $(-1, 1/a)$ $(0, 1)$ $(1, a)$

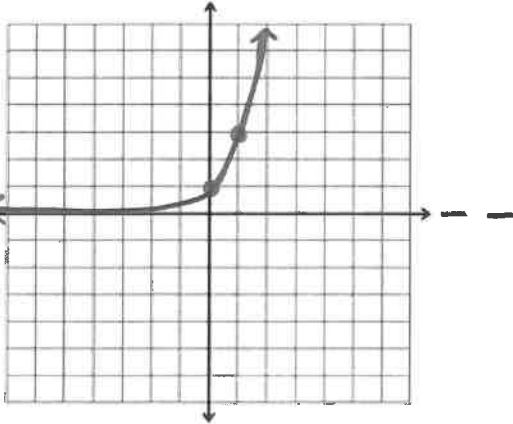
*a = base

2. Draw a sketch of each exponential function:

a. $f(x) = 3^x$

$a = 3$
HA $y = 0$

x	y
0	1
1	3

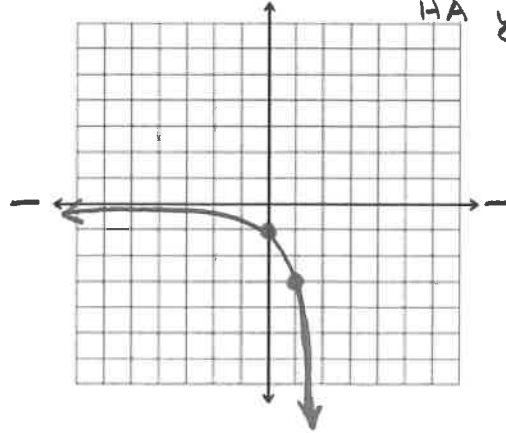


b. $f(x) = -3^x$

*reflect over x-axis

$a = 3$
HA $y = 3$

x	3^x	-3^x
0	1	-1
1	3	-3

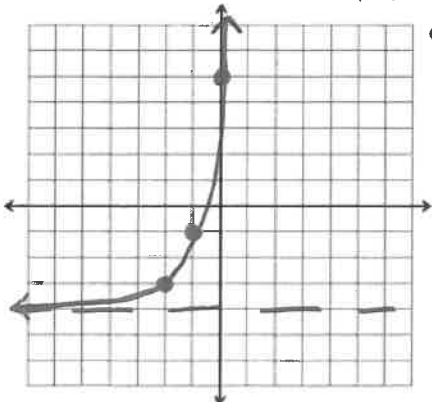


c. $f(x) = 3^{x+2} - 4$

$a = 3$
left 2

down 4 \rightarrow HA $y = -4$

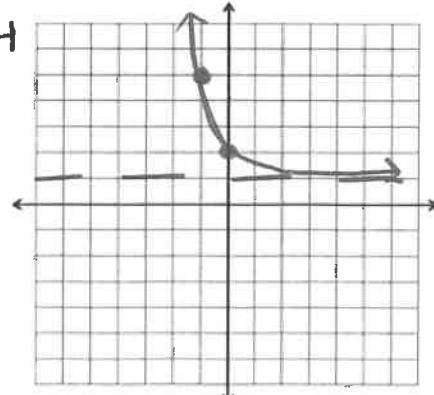
x	3^x
0	1
1	3



left 2	down 4
-2	-3
-1	-1

d. $f(x) = 3^{-x} + 1$

$a = 3$
reflect over y-axis
up 1 \rightarrow HA $y = 1$



*equivalent to $f(x) = (\frac{1}{3})^x + 1$ when $0 < a < 1$ decay

3. Solve the following exponential equations:

a. $5^x = \frac{1}{125}$

$$5^x = 5^{-3}$$

$$x = -3$$

b. $3^{x+1} = 9^{x-3}$

$$3^{x+1} = (3^2)^{x-3}$$

$$3^{x+1} = 3^{2x-6}$$

$$x+1 = 2x-6$$

$$x = 7$$

c. $x^{2/3} = 251$

$$x = 251^{3/2}$$

$$= \sqrt{251}^3$$

$$= 251\sqrt{251}$$

Compound Interest Formulas

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = initial investment

r = rate

t = time

$$A = Pe^{rt}$$

n = number of times compounded in year

4. Find the accumulated value of an investment of \$5000 for 10 years at an interest rate of 6.5% if the money is:

a. Compounded semiannually

$$P = 5000$$

$$t = 10$$

$$r = 0.065$$

$$n = 2$$

$$A = 5000\left(1 + \frac{0.065}{2}\right)^{2(10)}$$

$$= \$9479.19$$

b. Compounded monthly $n = 12$

$$A = 5000\left(1 + \frac{0.065}{12}\right)^{12(10)}$$

$$A = \$9560.92$$

5. Suppose you have \$6000 to invest. Which investment yields the greatest return over 4 years:

$$P = 6000$$

$$t = 4$$

a. 8.25% compounded quarterly

$$r = 0.0825$$

$$n = 4$$

$$A = 6000\left(1 + \frac{0.0825}{4}\right)^{4(4)}$$

$$= \$8317.84$$

b. 8.3% compounded semiannually

$$r = 0.083$$

$$n = 2$$

$$A = 6000\left(1 + \frac{0.083}{2}\right)^{2(4)}$$

$$= \$8290.70$$

c. 8.275% compounded continuously

$$A = 6000 e^{0.08275(4)}$$

$$= \$8354.16$$

6. The number of bacteria present in a culture can be modeled by the equation $B(t) = 10e^{0.483t}$, where t is the time in minutes.

a. Find $B(1)$.

$$B(1) = 10e^{0.483(1)}$$

$$\approx 16.209$$

b. What does this mean in context?

After 1 minute there are about 16 bacteria present in the culture

7. The 1986 explosion at the Chernobyl nuclear power plant in the former Soviet Union sent about 1000 kilograms of radioactive cesium-137 into the atmosphere. The function $f(x) = 1000(0.5)^{x/30}$ describes the amount, $f(x)$, in kilograms, of cesium-137 remaining in Chernobyl x years after 1986. If even 100 kilograms of cesium-137 remain in Chernobyl's atmosphere, the area is considered unsafe for human habitation.

a. Find $f(80)$.

$$f(80) = 1000(0.5)^{80/30}$$
$$= 157.49$$

b. What does this mean in practical terms?

80 years after 1986 so in the year 2066 there will be 157.49 kilograms of cesium-137 in Chernobyl's atmosphere, so the area will be unsafe for human habitation

extension:

In what year will Chernobyl be safe for human habitation?

$$100 = 1000(0.5)^{t/30}$$

* can't solve by hand yet

↳ need an inverse function

use calc "solve" function

$$t = 99.658$$

in 2086 Chernobyl will be safe for human habitation

