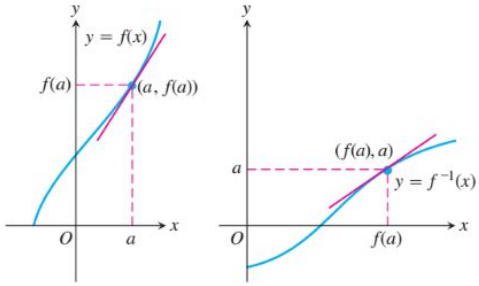
Let f and g be inverse functions. What is true about inverses?

Theorem 1 **Derivatives of Inverse Functions**

If f is differentiable at every point of an interval I and $f'(x)$ is never zero on I, then f has an inverse g, and g is differentiable at every point of the interval $f(I)$. If $g(x) = x$, the **inverse function slope relationship** relates the derivative by the equation

$$g'(x) = \frac{1}{f'(g(x))}$$



The slopes are reciprocal: $\left.\frac{df^{-1}}{dx}\right|_{f(a)} = \frac{1}{\left.\frac{df}{dx}\right|_a}$

1. Let f and g be function that are differentiable everywhere. If g is the inverse function of f and if $g(-2) = 5$ and $f'(5) = -\frac{1}{2}$, then $g'(-2) = ?$

2. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$. What is the value of $g'(3)$?
   a. $-\frac{1}{2}$
   b. $-\frac{1}{4}$
   c. $\frac{1}{6}$
   d. $\frac{1}{3}$
   e. The value of $g'(3)$ cannot be determined with the given information.

3. The function h is given by $h(x) = x^5 + 3x - 2$ and $h(1) = 2$. If h^{-1} is the inverse of h, what is the value of $(h^{-1})'(2)$?
   a. $\frac{1}{83}$
   b. $\frac{1}{8}$
   c. $\frac{1}{2}$
   d. 1
   e. 8

4. Let $f(x) = (2x + 1)^3$ and let $g(f(x)) = x$. Given that $f(0) = 1$, what is the value of $g'(1)$?
   a. $-\frac{2}{27}$
   b. $\frac{1}{54}$
   c. $\frac{1}{27}$
   d. $\frac{1}{6}$
   e. 6

5. Calc Active: If $f(x) = \sin x + 2x + 1$ and g is the inverse function of f, what is the value of $g'(1)$?
   a. $\frac{1}{3}$
   b. 1
   c. 3
   d. $\frac{1}{2+\cos 1}$
   e. $2 + \cos 1$

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\cdot\sqrt{x^2-1}}$$
$$\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{|x|\cdot\sqrt{x^2-1}}$$
$$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

1. Find the derivative of y with respect to the appropriate variable.
   a. $y = \cos^{-1}(\frac{1}{x})$

   b. $y = s\sqrt{1-s^2} + \cos^{-1}s$

c. $x(t) = \tan^{-1}(t^2),\ t = 1$

d. $y = \sec^{-1}(5s)$

e. $y = \cot^{-1}\sqrt{t}$

f. $y = \csc^{-1}\frac{x}{2}$

2. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t)$. Find the velocity at the indicated value of t.
   a. $x(t) = \sin^{-1}(\frac{\sqrt{t}}{4}),\ \ t = 4$