

### Definition of the Logarithmic Function

For  $x > 0$  and  $b > 0, b \neq 1$ ,

$$y = \log_b x \text{ is equivalent to } b^y = x$$

The function  $y = \log_b x$  is a **logarithmic function with base  $b$**

Logarithmic	Exponential
	$3^4 = 81$
$\log_{1/2} 8 = -3$	
	$10^3 = 1000$
$\log_5 \frac{1}{125} = -3$	
	$12^1 = 12$
$\log_6 1 = 0$	

### Properties of Logarithms

$$\star \log_b 1 =$$

$$\star \log_b b =$$

$$\star \log_b b^x =$$

$$\star b^{\log_b x} =$$

1. Evaluate each expression without a calculator:

a.  $\log_7 49 =$

b.  $\log_3 27 =$

c.  $\log_6 \sqrt{6} =$

d.  $\log_6 1 =$

e.  $\log_3 \frac{1}{9} =$

f.  $\log_{81} 9 =$

g.  $\log_{11} 11 =$

h.  $\log_4 4^6 =$

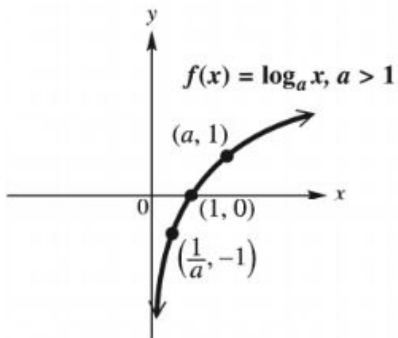
Exponential functions and logarithmic functions are \_\_\_\_\_

2. If  $f(x)$  and  $g(x)$  are inverse functions, determine the missing function:

a.  $f(x) = 5^x$  and  $g(x) =$  \_\_\_\_\_

b.  $f(x) = \underline{\hspace{2cm}}$  and  $g(x) = \log_7 x$

Parent Graph:



Domain:  $\underline{\hspace{2cm}}$

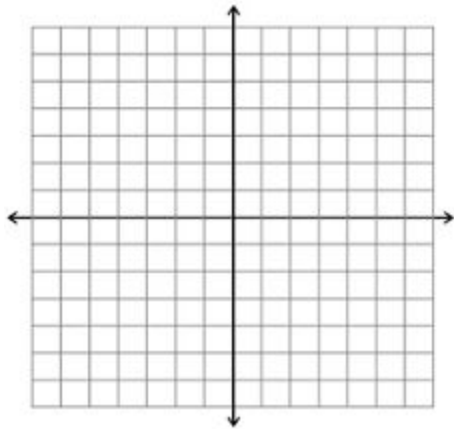
Range:  $\underline{\hspace{2cm}}$

Vertical Asymptote:  $\underline{\hspace{2cm}}$

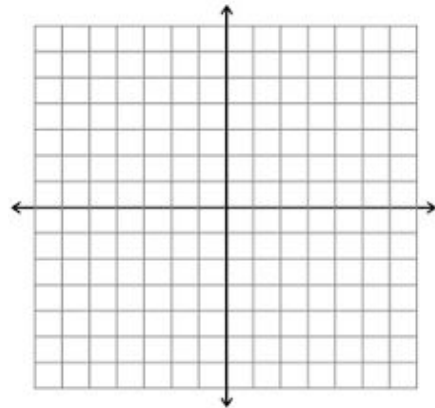
Key points:  $\underline{\hspace{4cm}}$

3. Draw a sketch of each function:

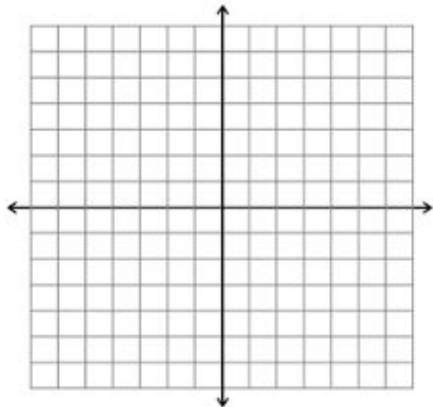
a.  $f(x) = \log_5 x$



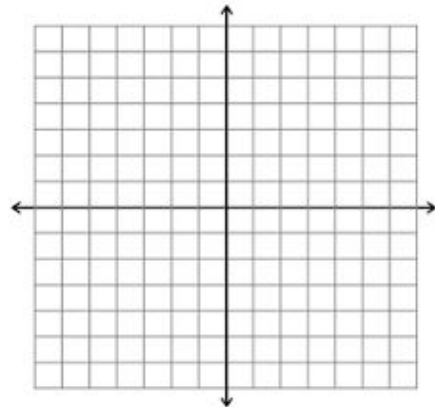
b.  $f(x) = \log_2(x + 2)$



c.  $f(x) = \log_4 x + 2$



d.  $f(x) = \log_2(x - 3) + 2$



Common Logarithms

Natural Logarithm

Logarithms with base 10

Logarithms with base  $e$

$\log_{10}x =$ _____	$\log_e x =$ _____
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4. Evaluate each expression without a calculator:

a.  $\log 1000 =$     b.  $\log 10^8 =$     c.  $10^{\log 33} =$     d.  $\ln e^6 =$     e.  $\ln \frac{1}{e^7} =$     f.  $e^{\ln 300} =$

5. Simplify each expression:

a.  $\ln e^{13x}$                                       b.  $10^{\log \sqrt[3]{x}}$                                       c.  $e^{\ln 7x^2}$

**Properties of Logarithms:**

The Product Rule	$\log_b(MN) = \log_b M + \log_b N$
The Quotient Rule	$\log_b \left( \frac{M}{N} \right) = \log_b M - \log_b N$
The Power Rule	$\log_b M^p = p \log_b M$

6. (Beginner) Use properties of logarithms to expand each logarithmic expression. Where possible, evaluate logarithmic expressions without a calculator.

a.  $\log(mn^3)$                                       b.  $\log \frac{u^4}{v}$                                       c.  $\log(ab)^2$

d.  $\log \frac{1}{z^3}$                                       e.  $\ln \frac{\sqrt{xy^4}}{z^5}$

7. (Intermediate) Use properties of logarithms to expand each logarithmic expression. Where possible, evaluate logarithmic expressions without a calculator.

a.  $\log_8 \frac{64}{\sqrt{x+1}}$

b.  $\ln \sqrt{ex}$

c.  $\log \frac{x}{1000}$

8. (Advanced) Use properties of logarithms to expand each logarithmic expression. Where possible, evaluate logarithmic expressions without a calculator.

$$\ln \left[ \frac{x^4 \sqrt{x^2+3}}{(x+3)^5} \right]$$

9. Condense the logarithmic expressions using properties of logarithms. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

a.  $\log 250 + \log 4$

b.  $\log_3 405 - \log_3 5$

c.  $5\log_b x + 6\log_b y$

d.  $2 \ln x - \frac{1}{2} \ln y$

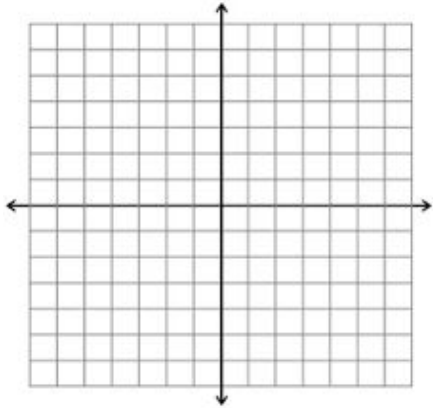
e.  $4 \ln x + 7 \ln y - 3 \ln z$

f.  $\frac{1}{3}(\log_4 x - \log_4 y)$

More Graphing!

10. Without a calculator, sketch the following logarithmic functions. State the range and domain of each function, intercepts (where possible without a calculator), and one additional point on the graph.

a.  $f(x) = \log_5(x + 6)$



Domain: \_\_\_\_\_

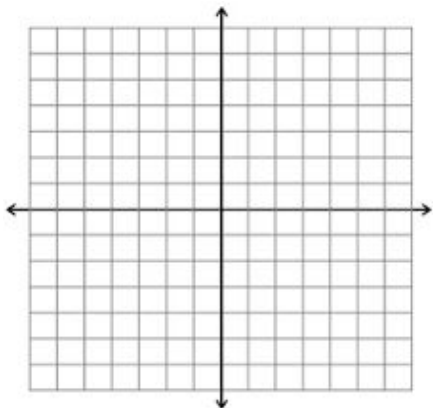
Range: \_\_\_\_\_

Intercept: \_\_\_\_\_

Asymptote: \_\_\_\_\_

Point(s): \_\_\_\_\_

b.  $f(x) = 2 + \log_5 x$



Domain: \_\_\_\_\_

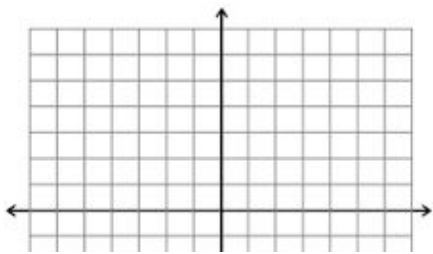
Range: \_\_\_\_\_

Intercept: \_\_\_\_\_

Asymptote: \_\_\_\_\_

Point(s): \_\_\_\_\_

c.  $f(x) = -\ln x + 2$



Domain: \_\_\_\_\_

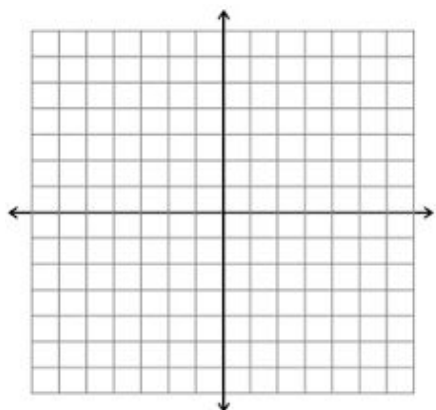
Range: \_\_\_\_\_

Intercept: \_\_\_\_\_

Asymptote: \_\_\_\_\_

Point(s): \_\_\_\_\_

d.  $f(x) = (\ln x - 7)^2$



Domain: \_\_\_\_\_

Range: \_\_\_\_\_

Intercept: \_\_\_\_\_

Asymptote: \_\_\_\_\_

Point(s): \_\_\_\_\_