tricks:
1) base of log stays in base of exponential and equal → exponent
OR
2) $\log_b x = y$ draw an "e"

Definition of the Logarithmic Function
For $x > 0$ and $b > 0$, $b \neq 1$,

★ log hidden exponential

$y = \log_b x$ is equivalent to $b^y = x$

$\log_3 9$ i.e. 3 to what power is 9

The function $y = \log_b x$ is a **logarithmic function with base** b

Logarithmic	Exponential
$\log_3 81 = 4$	$3^4 = 81$
$\log_{1/2} 8 = -3$	$(\frac{1}{2})^{-3} = 8$
$\log_{10} 1000 = 3$	$10^3 = 1000$
$\log_5 \frac{1}{125} = -3$	$5^{-3} = \frac{1}{125}$
$\log_{12} 12 = 1$	$12^1 = 12$
$\log_6 1 = 0$	$6^0 = 1$

Properties of Logarithms

★ $\log_b 1 = 0$ why: $b^{?} = 1$

★ $\log_b b = 1$ why: $b^{?} = b$

★ $\log_b b^x = x$ why: $b^{?} = b^x$

★ $b^{\log_b x} = x$

1. Evaluate each expression without a calculator:

a. $\log_7 49 = 2$
b. $\log_3 27 = 3$
c. $\log_6 \sqrt{6} = 1/2$
d. $\log_6 1 = 0$
e. $\log_3 \frac{1}{9} = -2$
f. $\log_{81} 9 = 1/2$
g. $\log_{11} 11 = 1$
h. $\log_4 4^6 = 6$

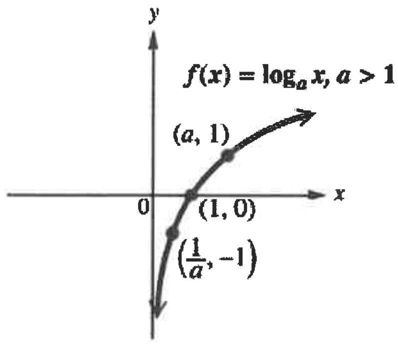
Exponential functions and logarithmic functions are inverse functions

2. If $f(x)$ and $g(x)$ are inverse functions, determine the missing function:

a. $f(x) = 5^x$ and $g(x) =$ $\log_5 x$

b. $f(x) =$ 7^x and $g(x) = \log_7 x$

Parent Graph:



Domain: $(0, \infty)$

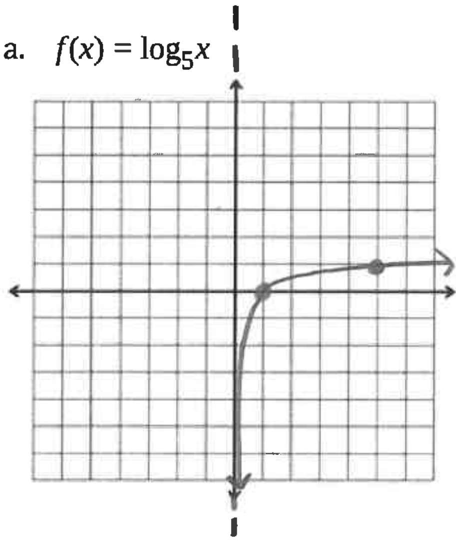
Range: $(-\infty, \infty)$

Vertical Asymptote: $x = 0$

Key points: $(1/a, -1)$ $(1, 0)$ $(a, 1)$

3. Draw a sketch of each function:

a. $f(x) = \log_5 x$

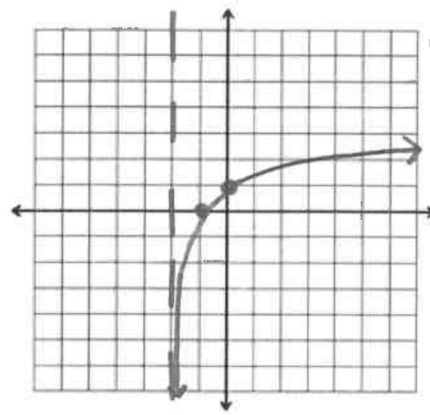


VA at $x = 0$

$(1, 0)$

$(5, 1)$

b. $f(x) = \log_2(x + 2)$ $\leftarrow 2$ VA $x = -2$



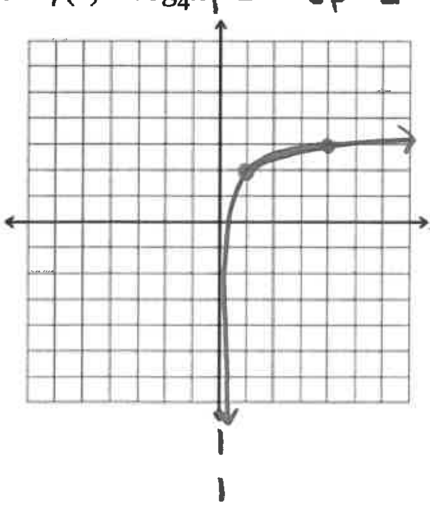
$\log_2 x$

$(1, 0)$ $(2, 1)$

$\log_2(x+2)$

$(-1, 0)$ $(0, 1)$

c. $f(x) = \log_4 x + 2$ up 2



VA $x = 0$

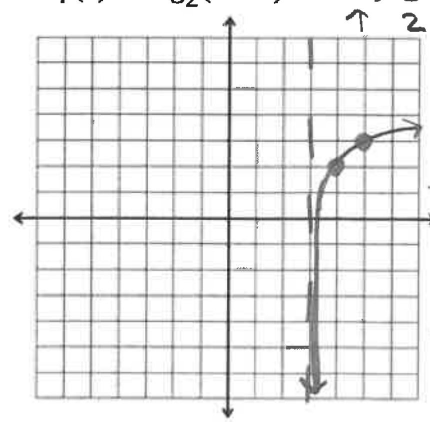
$\log_4 x$

$(1, 0)$ $(4, 1)$

$\log_4 x + 2$

$(1, 2)$ $(4, 3)$

d. $f(x) = \log_2(x - 3) + 2$ $\rightarrow 3$ $\uparrow 2$ VA $x = 3$



$\log_2 x$

$(1, 0)$ $(2, 1)$

$\log_2(x-3)$

$(4, 0)$ $(5, 1)$

$\log_2(x-3)+2$

$(4, 2)$ $(5, 3)$

Common Logarithms	Natural Logarithm
Logarithms with base 10 $\log_{10} x =$ log x	**Logarithms with base *e*** $\log_e x =$ ln x

4. Evaluate each expression without a calculator:

a. $\log 1000 =$ 3 b. $\log 10^8 =$ 8 c. $10^{\log 33} =$ 33 d. $\ln e^6 =$ 6 e. $\ln \frac{1}{e^7} = -7$ f. $e^{\ln 300} = 300$

$\ln e^{-7}$

5. Simplify each expression:

a. $\ln e^{13x}$ $13x$ b. $10^{\log \sqrt[3]{x}}$ $\sqrt[3]{x}$ c. $e^{\ln 7x^2}$ $7x^2$

Properties of Logarithms:

The Product Rule	$\log_b(MN) = \log_b M + \log_b N$
The Quotient Rule	$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$
The Power Rule	$\log_b M^p = p\log_b M$

6. (Beginner) Use properties of logarithms to expand each logarithmic expression. Where possible, evaluate logarithmic expressions without a calculator.

a. $\log(mn^3)$

$= \log m + \log n^3$

$= \log m + 3\log n$

b. $\log \frac{u^4}{v}$

$= \log u^4 - \log v$

$= 4\log u - \log v$

c. $\log(ab)^2$

$= 2\log ab$

$= 2(\log a + \log b)$

$= 2\log a + 2\log b$

d. $\log \frac{1}{z^3} = \log 1 - \log z^3$

$= 0 - 3\log z$

$= -3\log z$

e. $\ln \frac{\sqrt{x}y^4}{z^5} = \ln \sqrt{x}\,y^4 - \ln z^5$

$= \ln\sqrt{x} + \ln y^4 - \ln z^5$

$= \frac{1}{2}\ln x + 4\ln y - 5\ln z$

7. (Intermediate) Use properties of logarithms to expand each logarithmic expression. Where possible, evaluate logarithmic expressions without a calculator.

a. $\log_8 \frac{64}{\sqrt{x+1}}$

$= \log_8 64 - \log_8 \sqrt{x+1}$

$= 2 - \frac{1}{2}\log_8 (x+1)$

b. $\ln\sqrt{ex}$

$= \frac{1}{2}\ln ex$

$= \frac{1}{2}\ln e + \frac{1}{2}\ln x$

$= \frac{1}{2}(1) + \frac{1}{2}\ln x$

$= \frac{1}{2} + \frac{1}{2}\ln x$

c. $\log \frac{x}{1000}$

$= \log x - \log 1000$

$= \log x - 3$

8. (Advanced) Use properties of logarithms to expand each logarithmic expression. Where possible, evaluate logarithmic expressions without a calculator.

$\ln\left[\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right] = \ln x^4\sqrt{x^2+3} - \ln (x+3)^5$

$= \ln x^4 + \ln\sqrt{x^2+3} - 5\ln(x+3)$

$= 4\ln x + \frac{1}{2}\ln(x^2+3) - 5\ln(x+3)$

9. Condense the logarithmic expressions using properties of logarithms. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

a. $\log 250 + \log 4$

$= \log (250)(4)$

$= \log 1000$

$= 2$

b. $\log_3 405 - \log_3 5$

$= \log_3 \frac{405}{5}$

$= \log_3 81$

$= 4$

c. $5\log_b x + 6\log_b y$

$= \log_b x^5 + \log_b y^6$

$= \log_b x^5 y^6$

d. $2\ln x - \frac{1}{2}\ln y$

$= \ln x^2 - \ln\sqrt{y}$

$= \ln \frac{x^2}{\sqrt{y}}$

e. $4\ln x + 7\ln y - 3\ln z$

$= \ln x^4 + \ln y^7 - \ln z^3$

$= \ln x^4 y^7 - \ln z^3$

$= \ln \frac{x^4 y^7}{z^3}$

f. $\frac{1}{3}(\log_4 x - \log_4 y)$

$= \frac{1}{3}\log_4 \frac{x}{y}$

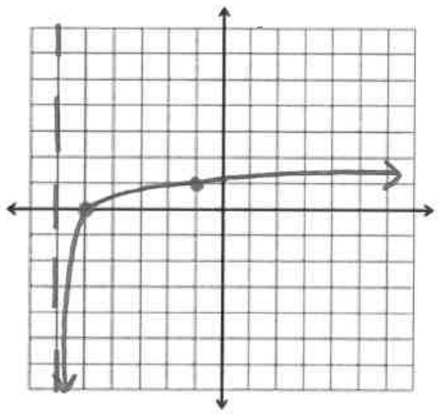
$= \log_4 \left(\frac{x}{y}\right)^{1/3}$

$= \log_4 \sqrt[3]{\frac{x}{y}}$

More Graphing!

10. Without a calculator, sketch the following logarithmic functions. State the range and domain of each function, intercepts (where possible without a calculator), and one additional point on the graph.

a. $f(x) = \log_5(x+6)$ ← 6



$\log_5 x$

(1, 0) (5, 1)

(-5, 0) (-1, 1)

Domain: (-6, ∞)

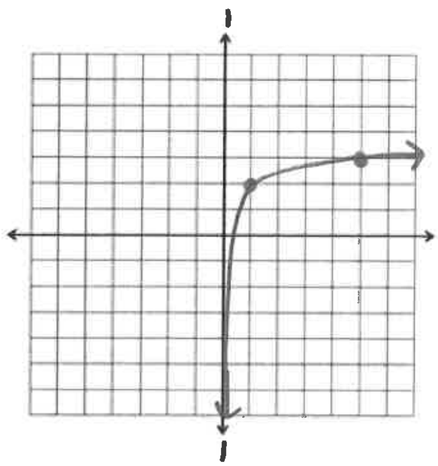
Range: (-∞, ∞)

Intercept: (-5, 0)

Asymptote: x = -6

Point(s): (-5, 0) (-1, 1)

b. $f(x) = 2 + \log_5 x$ ↑ 2



$\log_5 x$

(1, 0) (5, 1)

(1, 2) (5, 3)

Domain: (0, ∞)

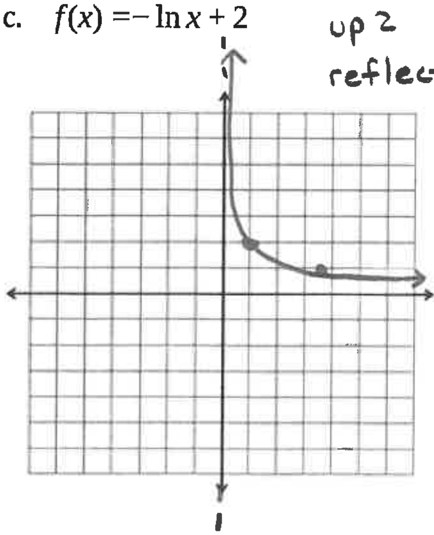
Range: (-∞, ∞)

Intercept:

Asymptote: x = 0

Point(s): (1, 2) (5, 3)

c. $f(x) = -\ln x + 2$ up 2
reflect x-axis



ln x

(1, 0) (e, 1)

-ln x

(1, 0) (e, -1)

-ln x + 2

(1, 2) (e, 1)

Domain: (0, ∞)

Range: (-∞, ∞)

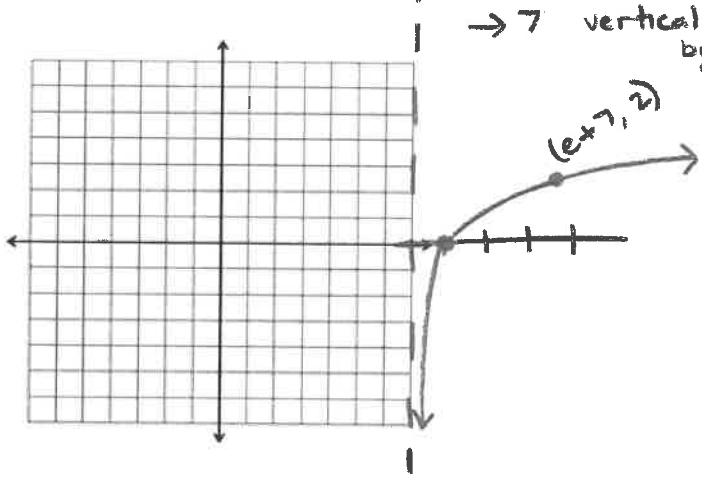
Intercept:

Asymptote: x = 0

Point(s): (1, 2) (e, 1)

d. $f(x) = \ln(x-7)^2 = 2\ln(x-7)$

$\rightarrow 7$ vertical stretch by 2



Domain: $(7, \infty)$

Range: $(-\infty, \infty)$

Intercept: $(8, 0)$

Asymptote: $x = 7$

Point(s): $(8, 0)$ $(e+7, 2)$

$\ln x$

$(1, 0)$ $(e, 1)$

$2\ln(x-7)$

$(8, 0)$ $(e+7, 2)$

$\uparrow$ stretch by 2