

AB Calculus
4.4 Derivatives of Exponential
and Logarithms

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \cdot \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}, \quad x \neq 0$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \cdot \ln a}, \quad x > 0$$

Examples:

1. Find y' if:

a. $y = e^{2x}$

$$y' = 2e^{2x}$$

d. $y = (\ln x)^2$

$$y' = \frac{2 \ln x}{x}$$

b. $y = x^2 e^x - x e^x$

$$y' = 2x e^x + x^2 e^x - (e^x + x e^x)$$

$$= 2x e^x + x^2 e^x - e^x - x e^x \quad e. \quad y = \frac{1}{\log_2 x}$$

$$= x e^x + x^2 e^x - e^x$$

$$= e^x(x^2 + x - 1)$$

$$y = (\log_2 x)^{-1}$$

$$y' = -(\log_2 x)^{-2}$$

$$\frac{}{x \ln 2}$$

$$= \frac{-1}{x \ln 2 (\log_2 x)^2}$$

c. $y = 9^{-x}$

$$y = 9^{-x} (-1) \ln 9$$

$$= -9^{-x} \ln 9$$

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f. $y = \log_3(1 + x \ln 3)$

$$\begin{aligned}y' &= \frac{1}{(1+x \ln 3) \ln 3} (\ln 3 + 0 \cdot x) \\&= \frac{\ln 3}{(1+x \ln 3) \ln 3} \\&= \frac{1}{1+x \ln 3}\end{aligned}$$

g. $y = x^{1+\sqrt{2}}$

$$\begin{aligned}y' &= (1+\sqrt{2})x^{1+\sqrt{2}-1} \\&= (1+\sqrt{2})x^{\sqrt{2}}\end{aligned}$$

2. Find $\frac{dy}{dx}$ if $y = e^{(x+x^2)}$

$$\begin{aligned}\frac{dy}{dx} &= e^{x+x^2} (1+2x) \\&= (1+2x)e^{x+x^2}\end{aligned}$$

3. At what point on the graph of the function $y = 2^t - 3$ does the tangent line have slope 21?

$$\frac{dy}{dt} = 2^t \ln 2$$

$$21 = 2^t \ln 2$$

$$\frac{21}{\ln 2} = 2^t$$

$$\ln\left(\frac{21}{\ln 2}\right) = \ln 2^t$$

$$\frac{\ln(21/\ln 2)}{\ln 2} = t$$

$$y = 2^{4.921} - 3$$

$$\approx 27.297$$

$(4.921, 27.297)$

$$t \approx 4.921$$

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4. Find the derivative of the following:

a. $y = x^{\sqrt{2}}$

$$y' = \sqrt{2} x^{\sqrt{2}-1}$$

b. $y = (2 + \sin 3x)^\pi$

$$\begin{aligned} y &= \pi (2 + \sin 3x)^{\pi-1} (\cos 3x)(3) \\ &= 3\pi \cos(3x) (2 + \sin 3x)^{\pi-1} \end{aligned}$$

4. Find y' of $y = x^x$

$$\ln y = x \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{x}{x} + \ln x$$

$$\frac{dy}{dx} = y(1 + \ln x)$$

$$= x^x(1 + \ln x)$$

5. Find $\frac{dy}{dx}$ of $y = x^{\sin(2x)}$

$$\ln y = \sin(2x) \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = 2 \cos(2x) \ln x + \frac{\sin 2x}{x}$$

$$\frac{dy}{dx} = x^{\sin(2x)} \left[\frac{2x \cos(2x) \ln x + \sin(2x)}{x} \right]$$

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1. The spread of a flu in a certain school is modeled by

$$P(t) = \frac{100}{1+e^{3-t}}$$

where $P(t)$ is the total number of students infected t days after the flu was first noticed.
Many of them may already be well again at time t .

- a. Estimate the initial number of students infected by the flu.
- b. How fast is the flu spreading after 3 days?
- c. When will the flu spread at its maximum rate? What is this rate?

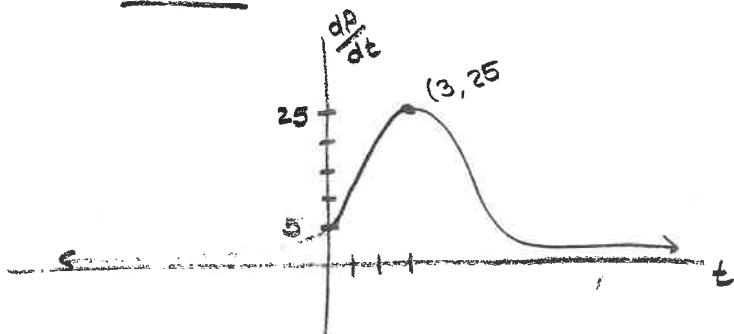
a) $P(0) = \frac{100}{1+e^{3-0}} = \frac{100}{1+e^3} \approx 5 \text{ students}$

b) $P'(t) = \frac{d}{dt} \left(100(1+e^{3-t})^{-1} \right)$
 $= -100(1+e^{3-t})^{-2} (e^{3-t})(-1)$

$$\frac{dp}{dt} = \frac{100 e^{3-t}}{(1+e^{3-t})^2} \quad \left. \frac{dp}{dt} \right|_{t=3} = 25 \text{ students per day}$$

- c) maximum on derivative graph

sketch



maximum rate occurs at about 3 days

when the flu is spreading at a rate of 25 students per day