2019-20

Honors Algebra 2 with Trig

4.4 Evaluating Logarithms and Change of Base

4.5 Solving Exponential and Logarithmic Equations

Change of Base Theorem

For any positive real numbers x, a and b, where $a \neq 1$ and $b \neq 1$, the following holds.

$$\log_a x = \frac{\log_b x}{\log_b a}$$

* any base b you want

1. Use the change-of-base theorem to find an approximation to four decimal places for each of the following:

   a. $\log_4 20 =$

   $\frac{\log 20}{\log 4} = 2.1609$

   b. $\log_2 0.7 =$

   $\frac{\ln 0.7}{\ln 2} = -0.5146$

2. Solve:

   a. $8^x = 21$

   $\log_8 8^x = \log_8 21$

   $x = \log_8 21$

   $x \approx 1.4641$

   b. $5^{2x+3} = 8^{x+1}$

   $\log_5 5^{2x+3} = \log_5 8^{x+1}$

   $2x+3 = (x+1)\log_5 8$

   $2x + 3 = x\log_5 8 + \log_5 8$

   $2x + x\log_5 8 = \log_5 8 - 3$

   $x(2 + \log_5 8) = \log_5 8 - 3$

   $x = \frac{\log_5 8 - 3}{\log_5 8 + 2} \approx -2.4125$

   c. $e^{|x|} = 50$

   $\ln e^{|x|} = \ln 50$

   $|x| = \ln 50$

   $x = \pm \ln 50$

   d. $e^{4x} \cdot e^{x-1} = 5e$

   $e^{4x+x-1} = 5e$

   $e^{5x-1} = 5e$

   $\ln e^{5x-1} = \ln 5e$

   $5x - 1 = \ln 5e$

   $x = \frac{1 + \ln 5e}{5}$

   $x = \frac{1 + \ln 5 + \ln e}{5}$

   $x = \frac{2 + \ln 5}{5}$

   $x \approx 0.7219$

   e. $e^{2x} - 6e^x + 5 = 0$

   $(e^x - 5)(e^x - 1) = 0$

   $e^x = 5$ $\quad e^x = 1$

   $x = \ln 5$ $\quad x = 0$

   $x = 0, \ln 5$

Recall that the domain of $y = \log_a x$ is $x > 0$. For this reason, *it is always necessary to check that proposed solutions of a logarithmic equation result in logarithms of positive numbers in the original equation.*

check for extraneous solutions!

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3. Solve the following and check for extraneous solutions.

a. $4\ln x = 36$

$\ln x = 9$

$\boxed{e^9 = x}$

b. $\log_3(x^3 - 5) = 1$

$3^1 = x^3 - 5$

$8 = x^3$ or $0 = x^3 - 8$

$\boxed{x = 2}$

$= (x-2)(x^2+2x+4)$

↓ prime & imaginary solutions

$0 > 2^2 - 4(1)(4)$

c. $\log(2x+1) + \log x = \log(x+8)$

$\log(2x+1)x = \log(x+8)$

$2x^2 + x = x + 8$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$ $x = -2$ extraneous

$\boxed{x = 2}$

d. $\log_3(4x+1)(x+1) = 3$

$3^3 = 4x^2 + 5x + 1$

$0 = 4x^2 + 5x - 26$

$0 = (x-2)(4x+13)$

$x = 2, -13/4$

↑ extraneous

$\boxed{x = 2}$

e. $\log_2(2x-5) + \log_2(x-3) = 3$

$\log_2(2x-5)(x-3) = 3$

$2^3 = 2x^2 - 11x + 15$

$0 = 2x^2 - 11x + 7$

$x = \dfrac{11 \pm \sqrt{121 - 4(2)(7)}}{4}$

$= \dfrac{11 \pm \sqrt{65}}{4}$

$\boxed{x = 4.7656}$ ~~0.7344~~

f. $\ln e^{\ln x} - \ln(x-4) = \ln 5$

$\ln x - \ln(x-4) = \ln 5$

$\ln \dfrac{x}{x-4} = \ln 5$

$\dfrac{x}{x-4} = 5$

$x = 5x - 20$

$4x = 20$

$\boxed{x = 5}$