

AB Calculus

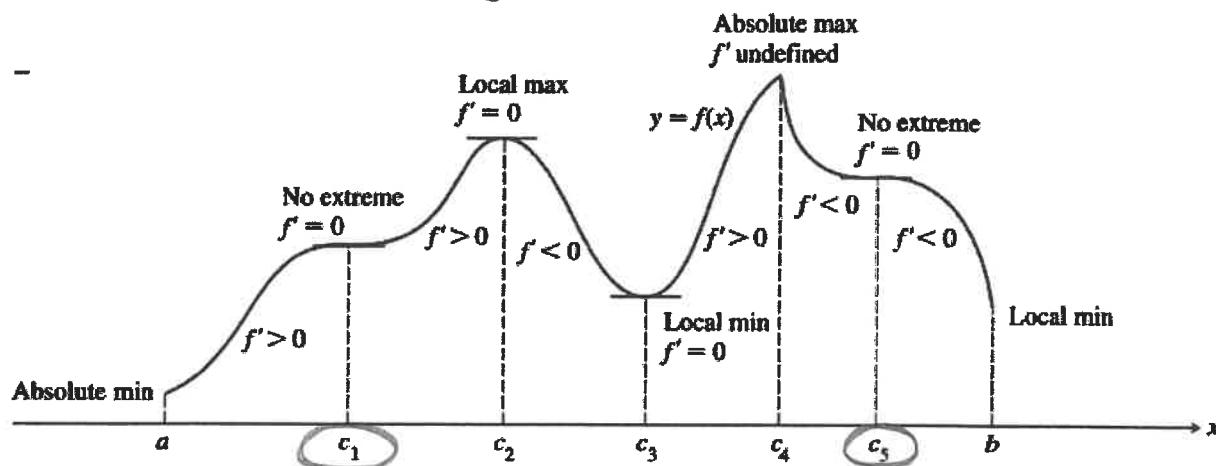
5.3 Connecting f' and f'' with the Graph of f

Finding critical points:

$$f'(x) = 0 \quad \text{or} \quad f'(x) = \text{und.}$$

Critical points don't immediately imply local extrema!

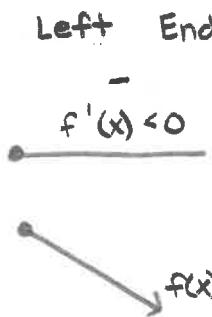
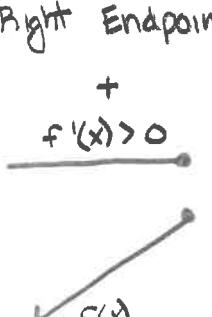
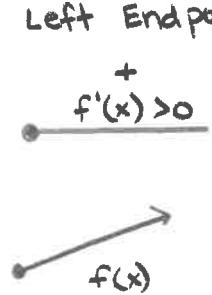
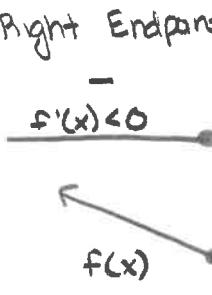
c_1 and c_5



First Derivative Test

Max in Interval	$\begin{array}{c c} + & - \\ f'(x) > 0 & f'(x) < 0 \\ \hline & \end{array}$ $f'(c) = 0$	$\begin{array}{c c} + & - \\ f'(x) > 0 & f'(x) < 0 \\ \hline & \end{array}$ $f'(c) = \text{und}$
		
Min in Interval	$\begin{array}{c c} - & + \\ f'(x) < 0 & f'(x) > 0 \\ \hline & \end{array}$ $f'(c) = 0$	$\begin{array}{c c} - & + \\ f'(x) < 0 & f'(x) > 0 \\ \hline & \end{array}$ $f'(c) = \text{und}$
		

AB Calculus
5.3 Connecting f' and f'' with the
Graph of f

Max at Endpoint	Left Endpoint  $f'(x) < 0$ $f(x)$	Right Endpoint  $f'(x) > 0$ $f(x)$
Min at Endpoint	Left Endpoint  $f'(x) > 0$ $f(x)$	Right Endpoint  $f'(x) < 0$ $f(x)$

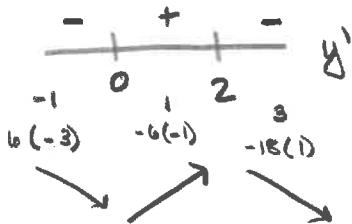
1. Use the first derivative test to determine the local extreme values of the function, and identify any absolute extrema. Support your answers graphically.

a. $y = -2x^3 + 6x^2 - 3$

$y' = -6x^2 + 12x$

$0 = -6x(x-2)$

$x = 0, 2$



$y(0) = -3$

$y(2) = -16 + 24 - 3 = 5$

Local max of 5 @ $x=2$ b/c y' changes from pos to neg. @ $x=0$ b/c y' changes from neg to pos.

b. $y = \begin{cases} 3 - x^2, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$ $y' = \begin{cases} -2x, & x < 0 \\ 2x, & x \geq 0 \end{cases}$

$y' = 0 @ x=0$ y' und?

$\lim_{x \rightarrow 0^-} y' \neq \lim_{x \rightarrow 0^+} y'$

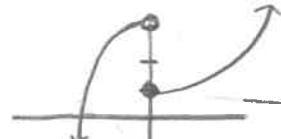
$3 \neq 1$

discontinuous @ $x=0$

y' und @ $x=0$

* always increasing

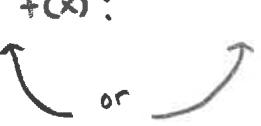
@ $x=0$ local max or min?



local min of 1
@ $x=0$

AB Calculus

5.3 Connecting f' and f'' with the
Graph of f

Concavity		
Concave Up	$f(x)$: 	when y' is increasing $y'' > 0$
Concave Down	$f(x)$: 	when y' is decreasing $y'' < 0$

2. Use the concavity test to determine the intervals on which the graph of the function is (a) concave up and (b) concave down.

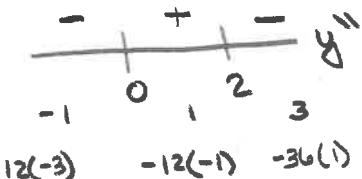
a. $y = -x^4 + 4x^3 - 4x + 1$

$$y' = -4x^3 + 12x^2 - 4$$

$$y'' = -12x^2 + 24x$$

$$0 = -12x(x-2) \quad y'' \text{ never und}$$

$$x = 0, 2$$



b. $y = e^x, \quad 0 \leq x \leq 2\pi$

$$y' = e^x$$

$$y'' = e^x$$

$$0 = e^x$$

no x -values y'' never und

$$y'' = e^x > 0$$

concave up
 $[0, 2\pi]$

concave up $(0, 2\pi)$
 concave down $(-\infty, 0) \cup (2, \infty)$

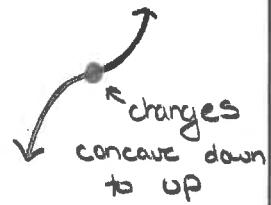
* tangent line implies $f(x)$ differentiable
which implies $f(x)$ continuous

AB Calculus
5.3 Connecting f' and f'' with the
Graph of f

Point of Inflection

A point where the graph of the function has a tangent line and where the concavity changes

* $f''(x) = 0$ and changes sign *
or und



3. Find all points of inflection of the function.

a. $y = x^3(4-x) = 4x^3 - x^4$

b. $y = \frac{x}{x+1}$

$y' = 12x^2 - 4x^3$

$y' = \frac{(x+1)(1) - x(1)}{(x+1)^2}$

$y'' = 24x - 12x^2$

$= \frac{x+1-x}{(x+1)^2}$

$0 = 12x(2-x)$

$= \frac{1}{(x+1)^2} = (x+1)^{-2}$

$x=0, 2$

$\begin{array}{c} - \\ \hline - & + & - \end{array} \quad y''$

poi @ $x=0$ and 2

b/c $y'' = 0$ @ $x=0, 2$

and y'' changes from
neg to pos @ $x=0$
and pos to neg @ $x=2$

$y'' = -2(x+1)^{-3}(1)$

$0 = \frac{-2}{(x+1)^3}$

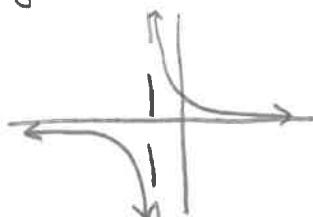
y'' und @
 $x=-1$

no x values

$y(-1) = \text{und}$

so no poi

* changes concavity @ $x=-1$
but y does not have a
tangent line @ $x=-1$



AB Calculus

5.3 Connecting f' and f'' with the
Graph of f

Second Derivative Test	
Max in Interval	1) $f'(c) = 0$ 2) $f''(c) < 0$ concave down
Min in Interval	1) $f'(c) = 0$ 2) $f''(c) > 0$ concave up

4. Find the local extreme values of $f(x) = x^3 - 12x - 5$.

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

$$0 = 3(x^2 - 4)$$

$$x = \pm 2$$

f' not und.

$$f''(2) = 12 > 0$$

$$f''(-2) = -12 < 0$$

local min @ $x=2$ b/c $f'(2)=0$ and $f''(2)>0$

local max @ $x=-2$ b/c $f'(-2)=0$ and $f''(-2)<0$

AB Calculus
5.3 Connecting f' and f'' with the
Graph of f

5. Let $f'(x) = 4x^3 - 12x^2$. (a) Identify where the extrema of f occur. (b) Find the intervals on which f is increasing and decreasing. (c) Find where the graph of f is concave up and concave down. (d) Sketch a possible graph of f .

$$f'(x) = 4x^3 - 12x^2 \quad f''(x) = 12x^2 - 24x$$

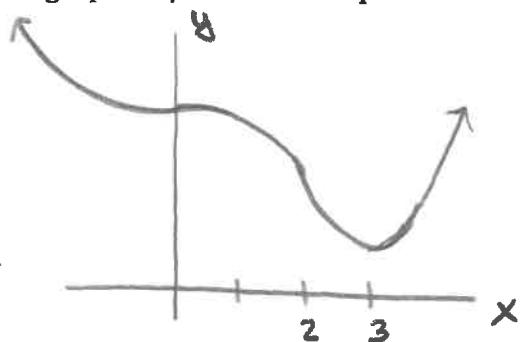
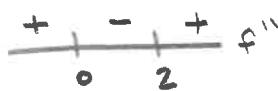
$$0 = 4x^2(x-3)$$

$$x=0, 3$$



$$0 = 12x(x-2)$$

$$x=0, 2$$



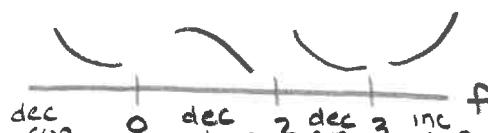
local min @ $x=3$

concave up $(-\infty, 0) \cup (2, \infty)$

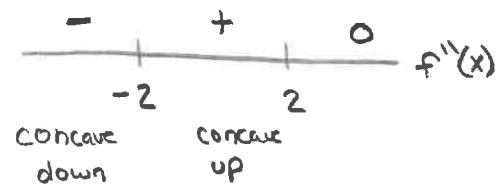
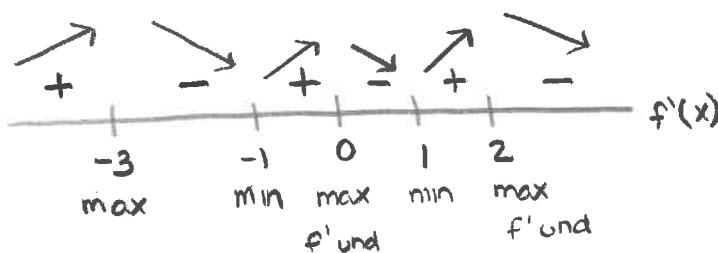
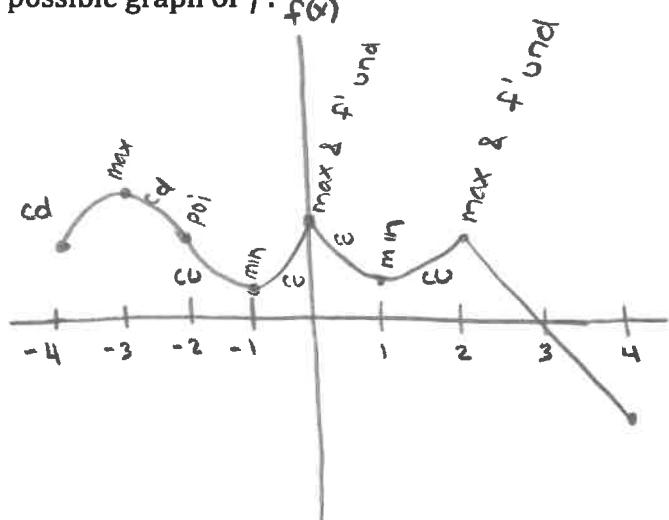
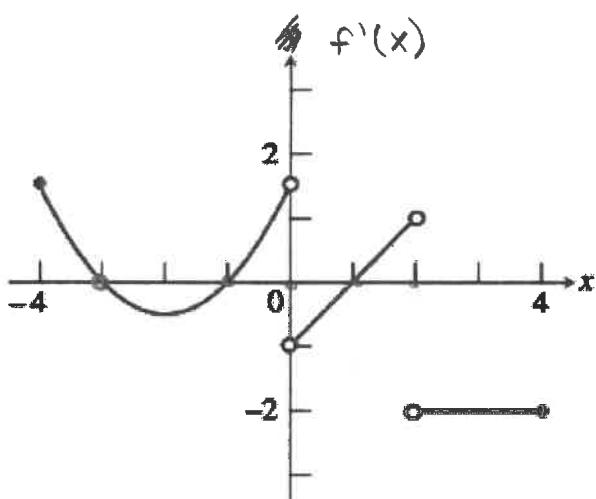
f dec. $(-\infty, 0) \cup (0, 3)$

concave down $(0, 2)$

f inc $(3, \infty)$

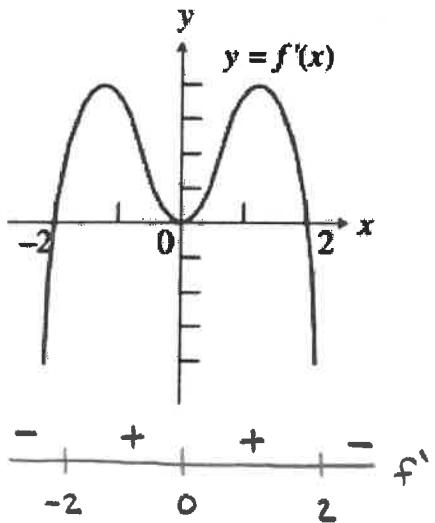


6. A function f is continuous on the interval $[-4, 4]$. The discontinuous function f' , with domain $[-4, 0] \cup (0, 2) \cup (2, 4]$, is shown below. (a) Find the x-coordinates of all local extrema and points of inflection of f . (b) Sketch a possible graph of f . $f(x)$



AB Calculus
5.3 Connecting f' and f'' with the
Graph of f

8. Use the graph of the function f' to estimate the intervals on which the function f is (a) increasing or (b) decreasing. Also, (c) estimate the x-coordinates of all local extrema values.



inc $(-2, 0) \cup (0, 2)$

dec $(-\infty, -2) \cup (2, \infty)$

local max @ $x = 2$

local min @ $x = -2$

