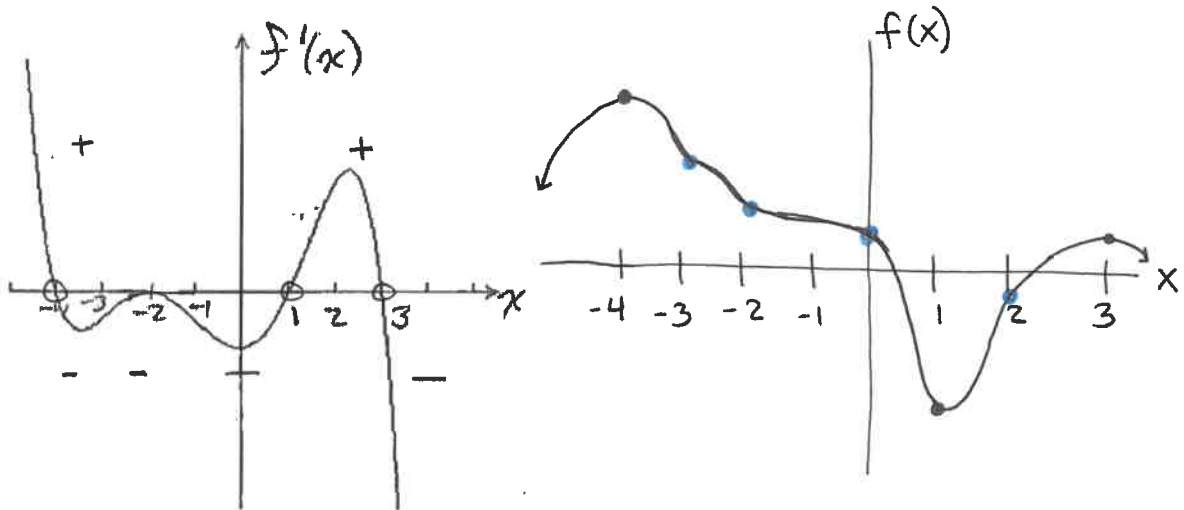


1. Below is the graph of the derivative of the function  $f$ . Draw a possible graph of the function  $f$ .



extrema max :  $x = -4, 3$   
min :  $x = 1$

poi :  $x = -3, -2, 0, 2$

2. For the functions below find:

- Critical points and determine which points are critical points only rather than stationary.
- Find the intervals on which  $f$  is increasing and decreasing.
- Find the coordinates of all local extrema.
- Find the intervals on which the function is concave up and concave down.
- Find the coordinates of any points of inflection.

$$f(x) = x^3 - 2x^2 - 2$$

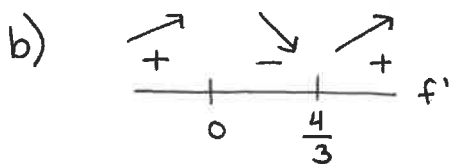
a)  $f'(x) = 3x^2 - 4x$

$$0 = 3x^2 - 4x$$

$$0 = x(3x - 4)$$

$$x = 0, \frac{4}{3}$$

both c.p & stationary



increasing  $(-\infty, 0)$  and  $(\frac{4}{3}, \infty)$

decreasing  $(0, \frac{4}{3})$

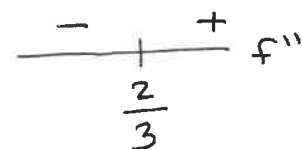
c) max at  $(0, -2)$

min at  $(\frac{4}{3}, -\frac{86}{27})$

d)  $f''(x) = 6x - 4$

$$0 = 6x - 4$$

$$\frac{4}{6} = \frac{2}{3} = x$$



concave up  $(\frac{2}{3}, \infty)$

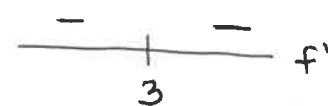
concave down  $(-\infty, \frac{2}{3})$

e) poi  $(\frac{2}{3}, -\frac{70}{27})$

$$f(x) = \frac{1}{x-3}$$

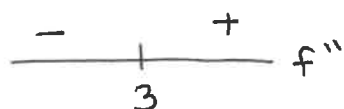
a)  $f'(x) = -(x-3)^{-2} \quad (1)$   
 $= -\frac{1}{(x-3)^2}$   
 $0 = -\frac{1}{(x-3)^2}$

c.p.  $x = 3$   
 not a stationary pt

b)   $f'$   
 dec  $(-\infty, 3) \cup (3, \infty)$

c) no extrema      e) no poi

d)  $f''(x) = 2(x-3)^{-3}$   
 $= \frac{2}{(x-3)^3}$

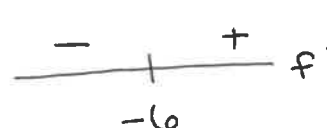
  $f''$

concave down  $(-\infty, 3)$   
 concave up  $(3, \infty)$

$$f(x) = (5x+30)^{\frac{2}{3}}$$

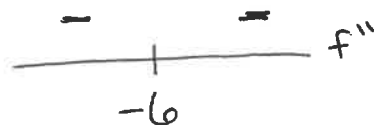
a)  $f'(x) = \frac{2}{3}(5x+30)^{-1/3}$   
 $0 = \frac{2}{3(5x+30)^{1/3}}$

c.p.  $x = -6$   
 not a stationary pt

b)   $f'$   
 increasing  $(-6, \infty)$   
 decreasing  $(-\infty, -6)$

c) min at  $(-6, 0)$

d)  $f''(x) = -\frac{2}{9}(5x+30)^{-4/3}$

  $f''$

concave down  $(-\infty, -6) \cup (-6, \infty)$

e) no poi

3. Sketch a continuous curve  $y = f(x)$  where  $f(1) = 0$ ,  $f'(0) = 0$ ,  $f'(2) = 0$ ,  $f''(x) < 0$  for  $x < 1$ , and  $f''(x) > 0$ , for  $x > 1$ .

concave up

c.p. c.p.  
slope is 0 at  $x=0$  slope 0 at  $x=2$

