IF	THEN
$f'(x) > 0$	f(x) is increasing
$f'(x) < 0$	f(x) is decreasing
$f'(x)$ is increasing	f(x) is concave up
$f'(x)$ is decreasing	f(x) is concave down
$f'(x)$ has a max or min	f(x) has a POI
$f'(x) = 0$ or D.N.E.	f(x) has a critical point
$f'(x)$ changes from + to -	f(x) has a max
$f'(x)$ changes from - to +	f(x) has a min
$f''(x) > 0$	f(x) is concave up
$f''(x) < 0$	f(x) is concave down
$f''(x) = 0$ and changes signs	f(x) has a POI
$f'(x) = 0$ AND $f'' < 0$	f(x) has a max
$f'(x) = 0$ AND $f'' > 0$	f(x) has a min

Therefore

$f'(x) > 0$ AND $f'' > 0$	f(x) is increasing & concave up
$f'(x) > 0$ AND $f'' < 0$	f(x) is increasing & concave down
$f'(x) < 0$ AND $f'' > 0$	f(x) is decreasing & concave up
$f'(x) < 0$ AND $f'' < 0$	f(x) is decreasing & concave down

5.1 Extreme Value of Functions
5.2 Mean Value Theorem
5.3 Connecting f' and f'' with the Graph of f
BC Calculus

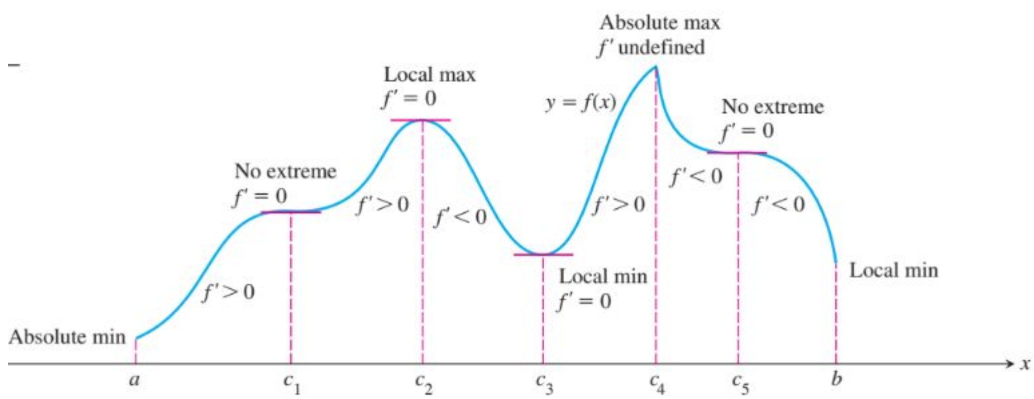
THEOREM 1 The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f has both a maximum value and a minimum value on the interval.

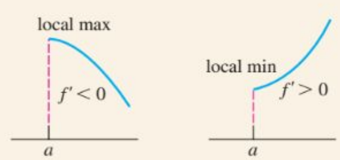
Critical Point: a point in the interior of the domain of a function f at which $f' = 0$ or f' does not exist is a critical point of f

Stationary Point: a point in the interior of the domain of a function f at which $f' = 0$ is called a stationary point of f

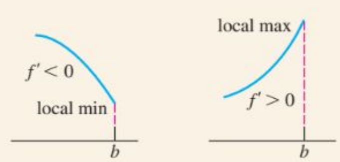
Critical points don't immediately imply local extrema!



At a left endpoint a:
If $f' < 0$ ($f' > 0$) for $x > a$, then f has a local maximum (minimum) value at a.



At a right endpoint b:
If $f' < 0$ ($f' > 0$) for $x < b$, then f has a local minimum (maximum) value at b.



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4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

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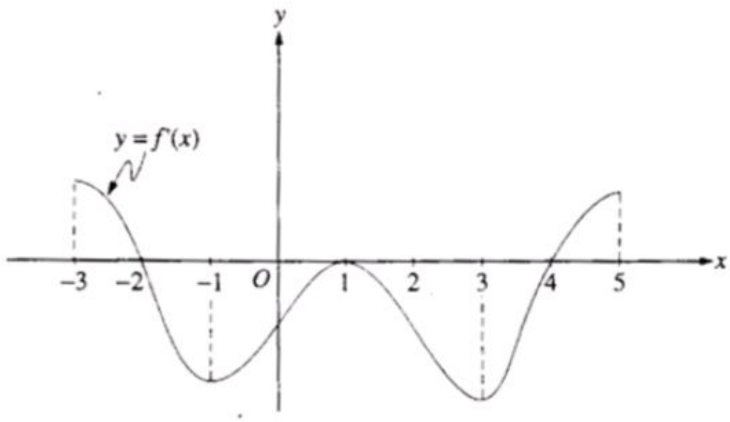

Note: This is the graph of the derivative of f, not the graph of f.

1. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

(a) For what values of x does f have a relative maximum? Why?

(b) For what values of x does f have a relative minimum? Why?

(c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

(d) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape the graph of the function f on the open interval $0 < x < 2$.

Note: The axes for this graph are provided in the pink booklet only.

5.1 Extreme Value of Functions
5.2 Mean Value Theorem
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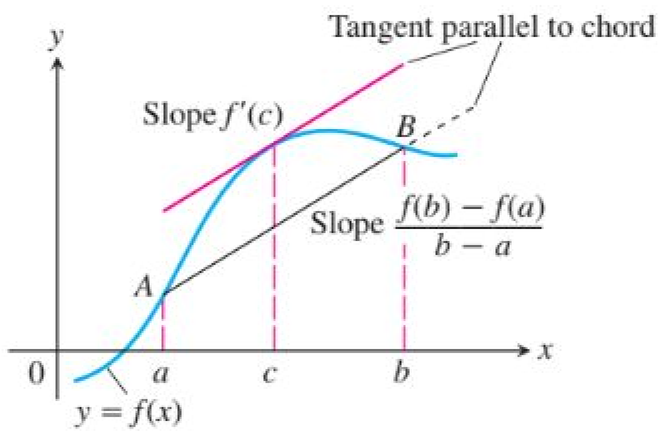
BC Calculus

THEOREM 3 Mean Value Theorem for Derivatives

If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Conditions of Theorem cannot be relaxed!



1. Find the value of c that satisfies the Mean Value Theorem on the interval $[-2, 1]$ for the function $f(x) = -\frac{x^2}{2} + x - \frac{1}{2}$.

2. Find the value of c that satisfies the Mean Value Theorem for $f(x) = \frac{x-1}{x}$ on $[1, 3]$

3. A trucker handed in a ticket at a toll booth showing that in 2 hours she had covered 159 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?

4. Determine if the Mean Value Theorem can be applied. If it can then find all values of c that satisfy the theorem. If it cannot, explain why not.

$$f(x) = \frac{-x^2}{4x+8}, \ [-3, -1]$$