IF	THEN
$f'(x) > 0$	$f(x)$ is increasing
$f'(x) < 0$	$f(x)$ is decreasing
$f'(x)$ is increasing	$f(x)$ is concave up
$f'(x)$ is decreasing	$f(x)$ is concave down
$f'(x)$ has a max or min	$f(x)$ has a POI
$f'(x) = 0$ or D.N.E.	$f(x)$ has a critical point
$f'(x)$ changes from + to -	$f(x)$ has a max
$f'(x)$ changes from - to +	$f(x)$ has a min
$f''(x) > 0$	$f(x)$ is concave up
$f''(x) < 0$	$f(x)$ is concave down
$f''(x) = 0$ and changes signs	$f(x)$ has a POI
$f'(x) = 0$ AND $f'' < 0$	$f(x)$ has a max
$f'(x) = 0$ AND $f'' > 0$	$f(x)$ has a min

Therefore

$f'(x) > 0$ AND $f'' > 0$	$f(x)$ is increasing & concave up
$f'(x) > 0$ AND $f'' < 0$	$f(x)$ is increasing & concave down
$f'(x) < 0$ AND $f'' > 0$	$f(x)$ is decreasing & concave up
$f'(x) < 0$ AND $f'' < 0$	$f(x)$ is decreasing & concave down

5.1 Extreme Value of Functions
5.2 Mean Value Theorem
5.3 Connecting f' and f" with the Graph of f
BC Calculus

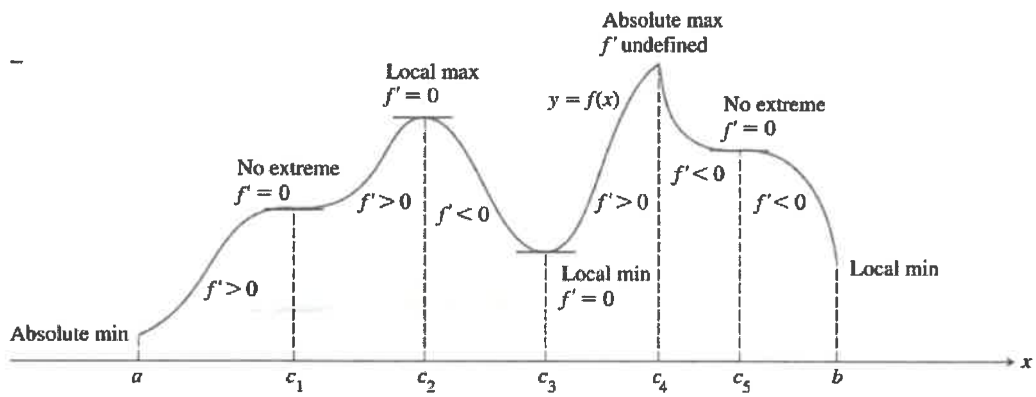
THEOREM 1 The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f has both a maximum value and a minimum value on the interval.

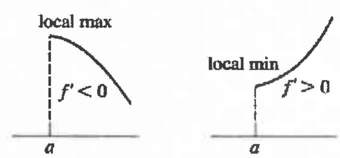
Critical Point: a point in the interior of the domain of a function f at which $f' = 0$ or f' does not exist is a critical point of f

Stationary Point: a point in the interior of the domain of a function f at which $f' = 0$ is called a stationary point of f

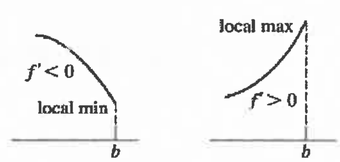
Critical points don't immediately imply local extrema!



At a left endpoint a:
If $f' < 0$ $(f' > 0)$ for $x > a$, then f has a local maximum (minimum) value at a.



At a right endpoint b:
If $f' < 0$ $(f' > 0)$ for $x < b$, then f has a local minimum (maximum) value at b.



2001 BC 4 No Calc

4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

a) $0 = h'(x) = \dfrac{x^2 - 2}{x}$

$x = \pm\sqrt{2}$

$h'(x)$ und at $x = 0$

$-$ $+$ $-$ $+$ $h'(x)$

$-\sqrt{2}$ 0 $\sqrt{2}$

horizontal tangents at $x = \pm\sqrt{2}$

local min at $x = \pm\sqrt{2}$ b/c $h'(x)$ changes sign from negative to positive

b) $h''(x) = \dfrac{x(2x) - (x^2 - 2)(1)}{x^2}$

$= \dfrac{2x^2 - x^2 + 2}{x^2}$

$= \dfrac{x^2 + 2}{x^2}$

$h''(x) > 0$ for all x except $x = 0$

so h is concave up for all $x \neq 0$

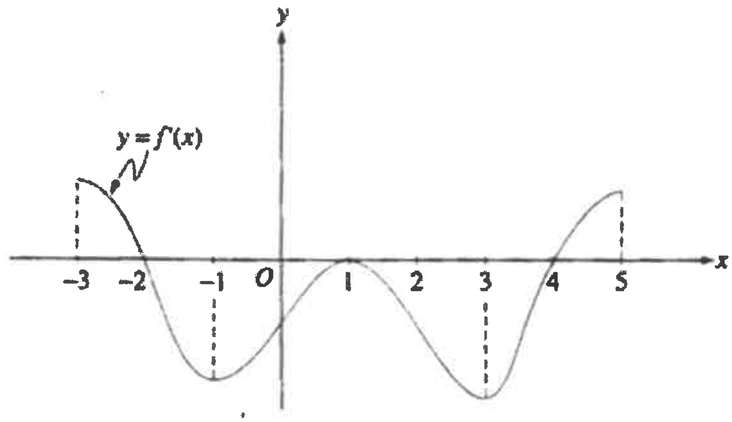
c) $h'(4) = \dfrac{16 - 2}{4} = \dfrac{7}{2}$

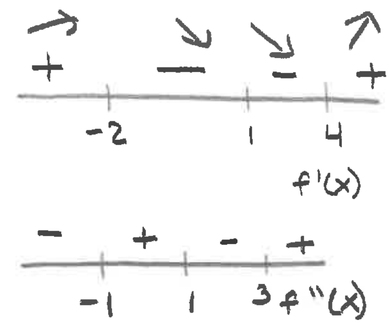
$y + 3 = 7/2(x - 4)$

d) below b/c h is concave up for $x > 4$

5.1 Extreme Value of Functions
5.2 Mean Value Theorem
5.3 Connecting f' and f'' with the Graph of f
BC Calculus

1996: AB-1





Note: This is the graph of the derivative of f, not the graph of f.

1. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.
   (a) For what values of x does f have a relative maximum? Why?
   (b) For what values of x does f have a relative minimum? Why?
   (c) On what intervals is the graph of f concave upward? Use f' to justify your answer.
   (d) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape the graph of the function f on the open interval $0 < x < 2$.
   Note: The axes for this graph are provided in the pink booklet only.

---

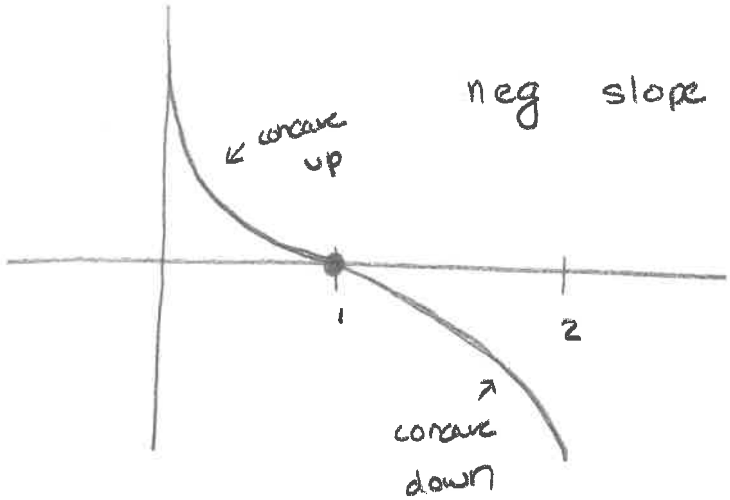
a) relative max at $x = -2$ b/c $f'(x)$ changes sign from positive to negative.

b) relative min at $x = 4$ b/c $f'(x)$ changes sign from negative to positive.

c) concave up on $(-1, 1) \cup (3, 5)$ b/c $f'(x)$ positive slope/increasing so $f''(x)$ is greater than 0.

d)

d)



neg slope $0<x<2$

5.1 Extreme Value of Functions
5.2 Mean Value Theorem
5.3 Connecting f' and f" with the Graph of f
BC Calculus

THEOREM 3 Mean Value Theorem for Derivatives

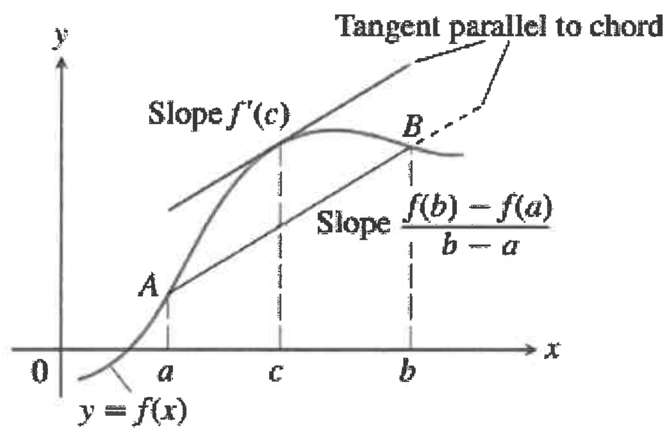
If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

★ continuous on closed

differentiable on open

Conditions of Theorem cannot be relaxed!



1. Find the value of c that satisfies the Mean Value Theorem on the interval $[-2, 1]$ for the function $f(x) =-\frac{x^2}{2} + x - \frac{1}{2}$.

$$f'(c) = \frac{f(1) - f(-2)}{1 - (-2)}$$

$$= \frac{0 - [-2 - 2 - 1/2]}{3}$$

$$= 3/2$$

$$f'(x) = -x + 1$$

$$\frac{3}{2} = -x + 1$$

$$\boxed{-1/2 = x}$$

2. Find the value of c that satisfies the Mean Value Theorem for $f(x) = \frac{x-1}{x}$ on $[1, 3]$

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

$$= \frac{2/3 - 0}{2}$$

$$= \frac{1}{3}$$

$$f'(x) = \frac{x(1) - (x-1)(1)}{x^2}$$

$$= \frac{x - x + 1}{x^2}$$

$$= 1/x^2$$

$$\frac{1}{3} = \frac{1}{x^2} \Rightarrow 3 = x^2 \Rightarrow \pm\sqrt{3} = x$$

$$\boxed{\sqrt{3} = x}$$

* in interval

3. A trucker handed in a ticket at a toll booth showing that in 2 hours she had covered 159 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?

$$f'(c) = \frac{159}{2} = 79.5 \text{ m/h}$$

by MVT at some point in the 2 hours her velocity was 79.5 mph, much greater than the speed limit

4. Determine if the Mean Value Theorem can be applied. If it can then find all values of c that satisfy the theorem. If it cannot, explain why not.

$$f(x) = \frac{-x^2}{4x+8}, \ [-3,-1]$$

$$x \neq -2$$

discontinuous and not differentiable w/n the interval [-3, -1]