

Absolute Extreme Values

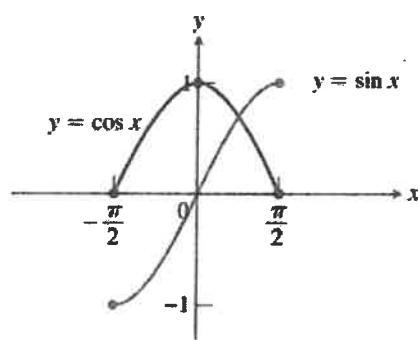
Let f be a function with Domain D . The $f(c)$ is the

- Absolute maximum value** on D if and only if $f(x) \leq f(c)$ for all x in D .
- Absolute minimum value** on D if and only if $f(x) \geq f(c)$ for all x in D .

Absolute (or Global) max and min values are also called absolute extrema

- Find the extreme values and where they occur:

a.



$$y = \cos x$$

abs max $(0, 1)$

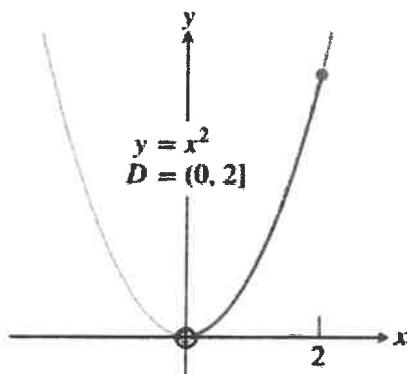
abs min $(-\frac{\pi}{2}, 0)$ and $(\frac{\pi}{2}, 0)$

$$y = \sin x$$

abs max $(\frac{\pi}{2}, 1)$

abs min $(-\frac{\pi}{2}, -1)$

b.



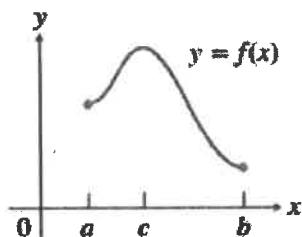
abs max $x = 2$

THEOREM 1 The Extreme Value Theorem (EVT)

If f is continuous on a closed interval $[a, b]$, then f has both a maximum value and a minimum value on the interval.

2. Identify each x value at which any absolute extrema value occurs. Explain how your answer is consistent with the Extreme Value Theorem.

a.



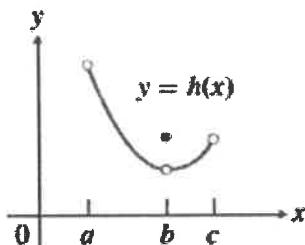
max at $x = c$

min at $x = b$

EVT does apply b/c

continuous and closed interval

b.



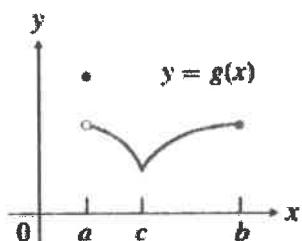
no max

no min

EVT doesn't apply b/c

discontinuous & open interval

c.

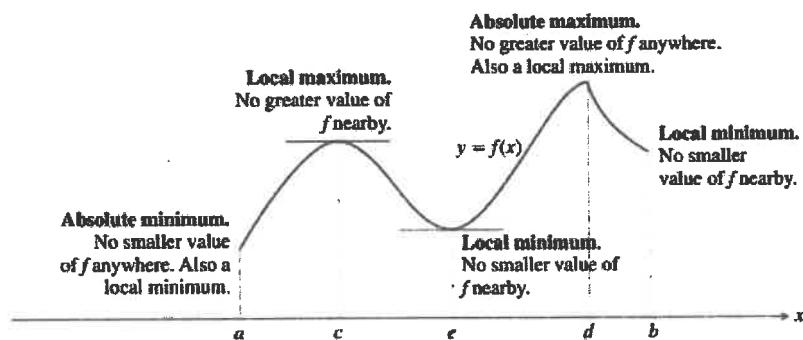


max at $x = a$

min at $x = c$

EVT doesn't apply b/c

discontinuous



Local Extreme Values

Let c be an interior point of the domain of the function f . Then $f(c)$ is a

a. Local Maximum Value at c if and only if $f(x) \leq f(c)$ for all x in some open interval containing c .

b. Local Minimum Value at c if and only if $f(x) \geq f(c)$ for all x in some open interval containing c .

Key: local max or min relative to nearby points!

Local Extrema also called Relative Extrema

An absolute extremum is also a local extremum.

Finding Extreme Values:

THEOREM 2 Local Extreme Values

If a function f has a local maximum value or a local minimum value at an interior point c of its domain, and if f' exists at c , then

$$f'(c) = 0.$$

Need to look at the following places to find local extrema:

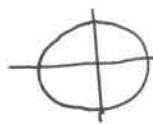
- 1. $f' = 0$
- 2. endpoints
- 3. f' does not exist

Critical Point: $f' = 0$ or f' DNE

Stationary Point: $f' = 0$

Stationary points and critical points are not necessarily the same!

\in in



AB Calculus
5.1 Extreme Value of Functions

3. Use analytic methods to find the extreme values of the function on the interval and where they occur. Identify any critical points that are not stationary points:

a. $f(x) = e^{-x}$, $-1 \leq x \leq 1$

$$f'(x) = -e^{-x}$$

② endpoints

① $f'(x) = 0 = -e^{-x}$

$$f(-1) = e^{-(-1)} = e$$

max of e at $x = -1$
min of $1/e$ at $x = 1$

none

$$f(1) = e^{-1} = 1/e$$

③ f' exists for all $x \in [-1, 1]$

b. $g(x) = \sec x$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$

$$g'(x) = \sec x \tan x$$

① $0 = \sec x \tan x$

$$0 = \sec x$$

$$0 = \tan x$$

$$0 = \frac{1}{\cos x}$$

no x values

③ g' DNE

$$x = \frac{\pi}{2}$$

makes g undefined

candidates

$$g(0) = 1$$

$$g(\pi) = -1$$

② endpoints

max of 1 at $x = 0$

min of -1 at $x = \pi$

both critical & stationary

$$g(-\frac{\pi}{2}) = \text{und}$$

$$g(\frac{3\pi}{2}) = \text{und}$$

4. Find the extreme values of the function and where they occur:

a. $f(x) = x^3 - 2x + 4$

$$f'(x) = 3x^2 - 2$$

$$0 = 3x^2 - 2$$

$$\pm \sqrt{2/3} = x$$

no endpoints

f' exists everywhere

$$f(\sqrt{2/3}) = 2.911$$

$$f(-\sqrt{2/3}) = 5.089$$

max $(-\sqrt{2/3}, 5.089)$

min $(\sqrt{2/3}, 2.911)$

$$y' = -1(x^2 - 1)^{-2}(2x)$$

$$0 = \frac{-2x}{(x^2 - 1)^2}$$

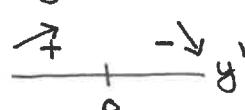
$$x = 0$$

$$x = \pm 1 \quad y' \text{ DNE}$$

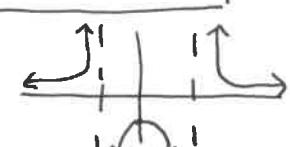
$$y(0) = -1$$

$$y(-1) = \text{und}$$

$$y(1) = \text{und}$$



max at $(0, -1)$



AB Calculus
5.1 Extreme Value of Functions

5. Identify the critical points and determine the local extreme values for the function below. Identify which critical points are not stationary points.

$$y = \begin{cases} 3 - x, & x < 0 \\ 3 + 2x - x^2, & x \geq 0 \end{cases}$$

$$y' = \begin{cases} -1, & x < 0 \\ 2 - 2x, & x \geq 0 \end{cases}$$

① $y' = 0$

$$0 = 2 - 2x$$

$$1 = x$$

y' und

$$\lim_{x \rightarrow 0^-} y' = -1 \quad \lim_{x \rightarrow 0^+} y' = 2$$

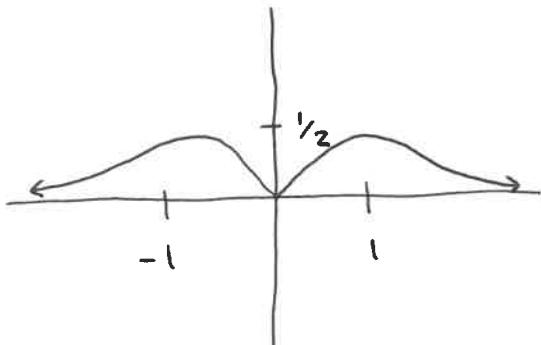
y' DNE at $x = 0$

$$y(0) = 3$$

$$y(1) = 3 + 2(1) - 1^2 = 4$$

6. Use graphical methods to find the extreme values of

$$f(x) = \left| \frac{x}{x^2+1} \right|, \quad -2 \leq x \leq 2$$



max at $(-1, \frac{1}{2})$
 max at $(1, \frac{1}{2})$
 min at $(0, 0)$

max at $(1, 4)$	$x=0, 1$ are critical pts
min at $(0, 3)$	$x=1$ stationary

