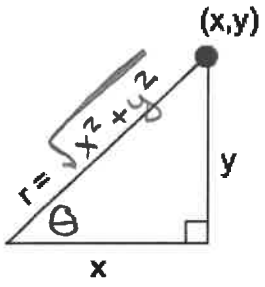
2019-20

Honors Algebra 2 with Trig

5.2 Trigonometric Functions

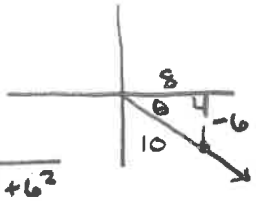
distance from point to origin is r



$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\quad \sec\theta = \frac{r}{x}$ $\quad \cot\theta = \frac{x}{y}$

1. The terminal side of an angle θ in standard position passes through the point $(8, -6)$. Find the values of the six trigonometric functions of angle θ.

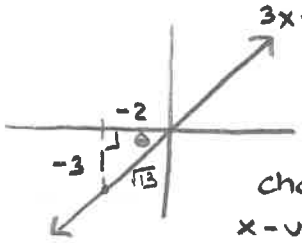


$r = \sqrt{8^2+6^2}$

$\sin\theta = -\frac{3}{5}$ $\quad \cos\theta = \frac{4}{5}$ $\quad \tan\theta = -\frac{3}{4}$

$\csc\theta = -\frac{5}{3}$ $\quad \sec\theta = \frac{5}{4}$ $\quad \cot\theta = -\frac{4}{3}$

2. Find the six trigonometric functions of the angle θ in standard position, if the terminal side of θ is defined by $3x - 2y = 0$, $x \le 0$.



$3x - 2y = 0$

$y = \frac{3}{2}x$

$x \le 0$

$= \sqrt{2^2+3^2}$

$= \sqrt{13}$

choose any x-value, $x \le 0$

* will reduce to same trig values

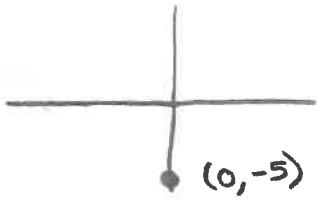
let $x = -2 \Rightarrow y = -3$

* rationalize denom

$\sin\theta = -\frac{3\sqrt{13}}{13}$ $\quad \cos\theta = -\frac{2\sqrt{13}}{13}$ $\quad \tan\theta = \frac{3}{2}$

$\csc\theta = -\frac{\sqrt{13}}{3}$ $\quad \sec\theta = -\frac{\sqrt{13}}{2}$ $\quad \cot\theta = \frac{2}{3}$

3. Find the six trigonometric functions for an angle θ in standard position with terminal side passing through $(0, -5)$.



$x = 0$ $\quad y = -5$ $\quad r = 5$

$\frac{-5}{5}$

$\sin\theta = -1$ $\quad \cos\theta = 0$ $\quad \tan\theta = \text{und}$

$\csc\theta = -1$ $\quad \sec\theta = \text{und}$ $\quad \cot\theta = 0$

Reciprocal Identities		
$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\tan\theta = \frac{1}{\cot\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

4. Find each function value:

a. $\tan\theta$, given that $\cot\theta = 4$

$\tan\theta = \frac{1}{4}$

b. $\sec\theta$, given that $\cos\theta = -\frac{2}{\sqrt{20}}$

$\sec\theta = -\frac{\sqrt{20}}{2} = -\sqrt{5}$

Signs of Trigonometric Function Values

Students (sin pos)	All (all pos)
Take (tan pos)	Calculus (cos pos)

x neg, y pos	x pos, y pos
x neg, y neg	x pos, y neg

sine pos, cosine neg, tangent neg	sine, cosine, tangent } pos
sine neg, cosine neg, tangent pos	sine neg, cos pos, tangent neg

5. Determine the signs of the trigonometric functions of an angle in standard position with the given measure

a. $260°$

$\sin\theta =$ neg	$\csc\theta =$ neg
$\cos\theta =$ neg	$\sec\theta =$ neg
$\tan\theta =$ pos	$\cot\theta =$ pos

b. $-60°$

$\sin\theta =$ neg	$\csc\theta =$ neg
$\cos\theta =$ pos	$\sec\theta =$ pos
$\tan\theta =$ neg	$\cot\theta =$ neg

6. Identify the quadrant (or possible quadrants) of an angle θ that satisfies the given condition: $\tan\theta > 0$, $\csc\theta < 0$

3rd quadrant

$\tan\theta > 0$ narrows to quad 1 and 3

$\csc < 0$ confirms quad 3

7. Proof time! See if you can determine what these identities are equal to:

Identities		
$\frac{\sin\theta}{\cos\theta} = \tan\theta$	$\frac{\cos\theta}{\sin\theta} = \cot\theta$	
$\sin^2\theta + \cos^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$

$3^2 = x^2 + (-\sqrt{2})^2$

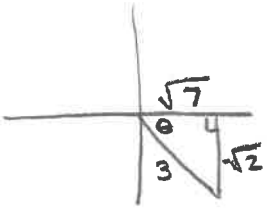
$7 = x^2$

8. Find $\cos\theta$ and $\tan\theta$, given that $\sin\theta = -\frac{\sqrt{2}}{3}$ and $\cos\theta > 0$

* 4th quad

$\sin\theta < 0$

$\cos\theta > 0$



$\cos\theta = \frac{\sqrt{7}}{3}$

$\tan\theta = -\frac{\sqrt{2}}{\sqrt{7}} = -\frac{\sqrt{14}}{7}$

$$\frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r} = \frac{y}{x} = \tan\theta$$ Q.E.D.

$$\frac{\cos\theta}{\sin\theta} = \frac{x/r}{y/r} = \frac{x}{y} = \cot\theta$$ Q.E.D.

$$\sin^2\theta + \cos^2\theta = (y/r)^2 + (x/r)^2$$

$$= \frac{y^2 + x^2}{r^2}$$

note $r = \sqrt{x^2 + y^2}$

$$= \frac{r^2}{r^2}$$

$$= 1$$

Q.E.D.

$$\tan^2\theta + 1 = (y/x)^2 + 1$$

$$= \frac{y^2 + x^2}{x^2}$$

note $r = \sqrt{x^2 + y^2}$

$$= \frac{r^2}{x^2}$$

$$= \left(\frac{r}{x}\right)^2 = \sec^2\theta$$ Q.E.D.

$$1 + \cot^2\theta = 1 + \left(\frac{x}{y}\right)^2$$

$$= 1 + \frac{x^2}{y^2}$$

$$= \frac{y^2 + x^2}{y^2}$$

note $r = \sqrt{x^2+y^2}$

$$= \frac{r^2}{y^2}$$

$$= \left(\frac{r}{y}\right)^2$$

$$= \csc^2\theta$$

Q.E.D.