Solving Triangles

To solve a triangle means to find the __measures__ *of all the angles and sides of the triangle.*

1. Solve right triangle ABC, if $B = 28°$ and $a = 25.3$ cm
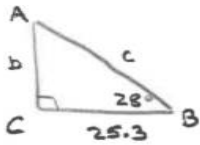
$\angle A = 62°$ $a = 25.3$ cm

$\angle B = 28°$ $b = 13.45$ cm

$\angle C = 90°$ $c = 28.65$ cm

$\tan 28 = \dfrac{b}{25.3}$ $\cos 28 = \dfrac{25.3}{c}$

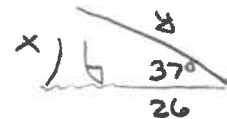$25.3 \tan 28 = b$ $c = \dfrac{25.3}{\cos 28}$

$13.45 = b$ $c = 28.65$

2. A tree snapped in high winds and bent over so that its top made an angle of $37°$ with the ground. If the top touched the ground 26 feet from the center of its base, how tall was the tree before it snapped and bent over? Give answer to the nearest tenth of a foot.

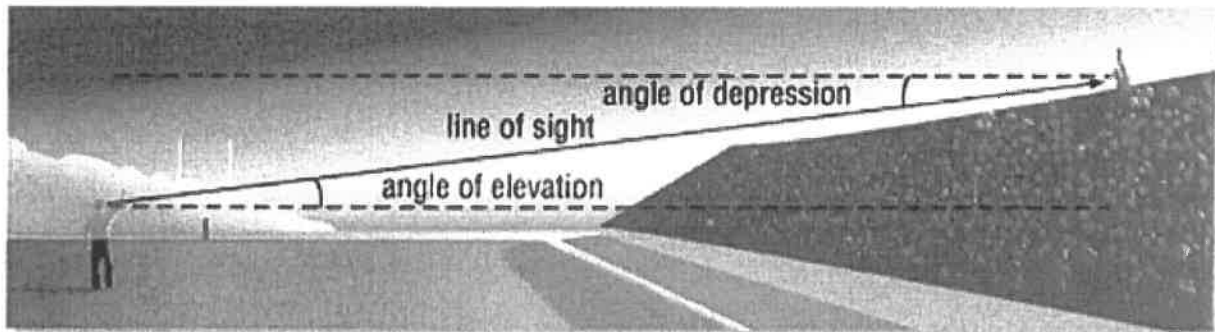$\cos 37 = \dfrac{26}{y}$ $\tan 37 = \dfrac{x}{26}$
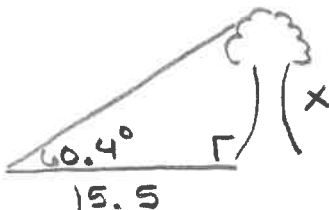
$y = \dfrac{26}{\cos 37}$ $19.5924 = x$

$y = 32.5555$ $x + y = \boxed{52.2 \text{ ft}}$

angle of depression

line of sight

angle of elevation

3. The angle of elevation from a point on the ground 15.5 m from the base of a tree to the top of the tree is $60.4°$. Find the height of the tree.
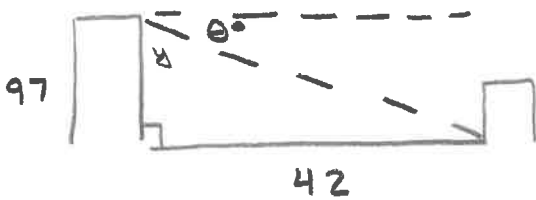
$\tan 60.4 = \dfrac{x}{15.5}$

$\boxed{27.2849 \text{ m} = \text{height}}$

4. Find the angle of depression from the top of a 97-ft building to the base of a building across the street located 42 ft away.
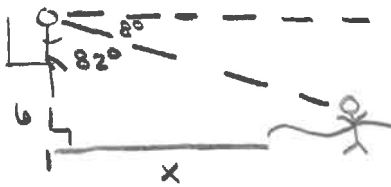
$\tan y = \dfrac{42}{97}$

$\theta = 90 - y$

$y = \tan^{-1}(42/97)$

$\boxed{\theta = 66.5879°}$

$y = 23.4121$

5. A lifeguard is watching a beach from a line of sight 6 feet above the ground. She sees a swimmer at an angle of depression of 8°. How far away from the tower is the swimmer?

$\tan 82 = \dfrac{x}{6}$

$\boxed{42.6922 \text{ ft} = x}$

6. To estimate the height of a tree she wants removed, Mrs. Long sights the tree's top at a 70° angle of elevation. She then steps back 10 meters and sights the top at a 26° angle. If mrs. Long's line of sight is 1.7 meters above the ground, how tall is the tree to the nearest meter?
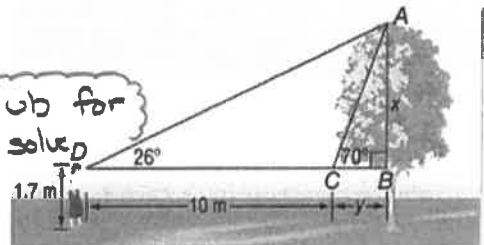
$\tan 70 = \dfrac{x}{y}$ $\tan 26 = \dfrac{x}{10+y}$

or sub for
y & solve
for x

$y \tan 70 = x$ $(10+y)\tan 26 = x$

$y \tan 70 = (10+y)\tan 26$

$y \tan 70 - y \tan 26 = 10 \tan 26$

$y(\tan 70 - \tan 26) = 10 \tan 26$

$y = \dfrac{10 \tan 26}{\tan 70 - \tan 26}$

$y = 2.15835$ *store in*
calc

$\tan 70 = \dfrac{x}{y}$

$x = 5.93$

$\text{tree} = 1.7 + 5.93$

$\boxed{= 7.63 \text{ m}}$

Bearing Method 2

Start with a north-south line and use an acute angle to show the direction, either east or west, from this line.

N
42°
N 42° E

31°
S
S 31° E

40°
S
S 40° W

N
52°
N 52° W
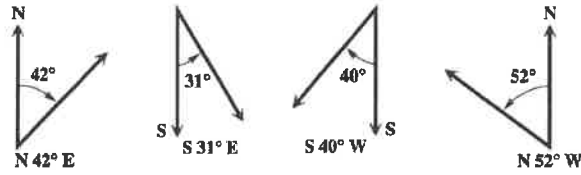
9. A ship leaves port and sails on a bearing of S 34° W for 2.5 hr. It then turns and sails on a bearing of N 56° W for 3.0 hr. If the ship's rate of speed is 18 knots (nautical miles per hour), find the distance that the ship is from port.

ship

port

34°

56°

ship ―――― x ―――― port

54   45

$56° + 34° = 90°$

↑

b/c alternate interior angles congruent
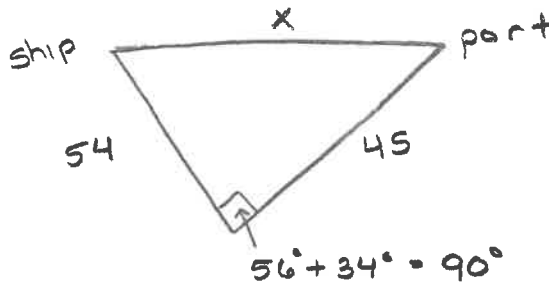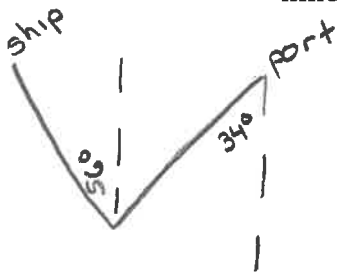
★ don't know port to ship directly west so can't say angle is $90 - 34 = 56°$

18 Knots · 2.5 hr = 45 mi

18 Knots · 3 hr = 54 mi

$$54^2 + 45^2 = x^2$$

$$\boxed{70.29 \text{ mi} = x}$$

7. Two buildings are sited from atop a 200-meter skyscraper. Building A is sited at a $35°$ angle of depression, while Building B is sited at a $36°$ angle of depression. How far apart are the two buildings to the nearest meter?
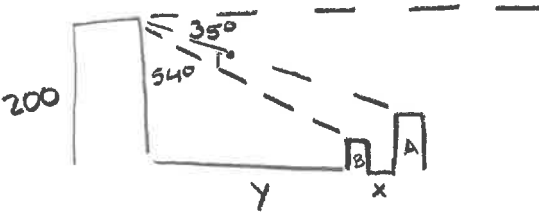
$$\tan 54 = \frac{y}{200} \qquad\qquad \tan 55 = \frac{x+y}{200}$$

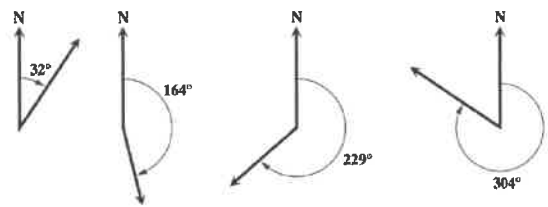$$200 \tan 54 = y \qquad\qquad 200 \tan(55) - x = y$$

$$200 \tan 54 = 200 \tan(55) - x$$

$$x = 200 \tan 54 - 200 \tan 55$$
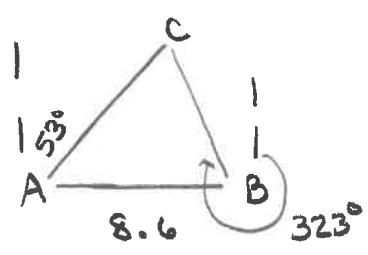
$$\boxed{x = 10 \text{ m}}$$

Bearing Method 1

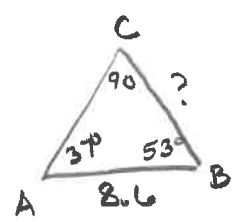When a single angle is given, it is understood that the bearing is measured in a clockwise direction from due north.

Bearings of 32°, 164°, 229°, and 304°

8. Radar stations A and B are on an east-west line, 8.6 km apart. Station A detects a plane at C, on a bearing of 53°. Station B simultaneously detects the same plane, on a bearing of 323°. Find the distance from B to C.

$$\cos 53 = \frac{BC}{8.6}$$

$$\boxed{BC = 5.1756 \text{ Km}}$$

$$m \angle C = 180 - 53 - 37$$

$$= 90°$$