DEFINITION Linearization

If f is differentiable at $x = a$, then the equation of the tangent line,

$$L(x) = f(a) + f'(a)(x - a),$$

defines the **linearization of f at a.** The approximation $f(x) \approx L(x)$ is the **standard linear approximation of f at a.** The point $x = a$ is the **center** of the approximation.

1. Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$, and use it to approximate $\sqrt{1.02}$ without a calculator. Then use a calculator to determine the accuracy of the approximation.

2. Find the linearization of $f(x) = \cos x$ at $x = \frac{\pi}{2}$ and use it to approximate $\cos 1.75$ without a calculator. Then use a calculator to determine the accuracy of the approximation.

Approximating Binomial Powers:

$$(1+x)^k \approx 1 + kx$$

Use this formula to find polynomials that will approximate the following functions for values of x close to 0:

a. $\sqrt[3]{1-x}$

b. $\frac{1}{1-x}$

c. $\sqrt{1+5x^4}$

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Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

~~(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your~~ answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?