DEFINITION Linearization

If f is differentiable at $x = a$, then the equation of the tangent line,

$$L(x) = f(a) + f'(a)(x - a),$$

defines the **linearization of f at a.** The approximation $f(x) \approx L(x)$ is the **standard linear approximation of f at a.** The point $x = a$ is the **center** of the approximation.

1. Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$, and use it to approximate $\sqrt{1.02}$ without a calculator. Then use a calculator to determine the accuracy of the approximation.

$L(x) = f(a) + f'(a)(x-a)$

$f'(x) = \frac{1}{2\sqrt{1+x}}$

$f'(0) = \frac{1}{2}$

$L(x) = 1 + \frac{1}{2}(x-0)$

$= 1 + \frac{1}{2}x$

$f(0) = \sqrt{1} = 1$

app $\sqrt{1.02}$

$x = 0.02$

$L(0.2) = 1 + \frac{1}{2}(0.02)$

$= 1.01$

Accuracy:

$\sqrt{1.02} = 1.00995049 4$

Error

$\frac{|1.009950494 - 1.01|}{1.009950494}$

$\approx 4.9 \times 10^{-5}$

2. Find the linearization of $f(x) = \cos x$ at $x = \frac{\pi}{2}$ and use it to approximate $\cos 1.75$ without a calculator. Then use a calculator to determine the accuracy of the approximation.

$f'(x) = -\sin x$

$f'(\pi/2) = -\sin \pi/2 =$

$f(\pi/2) = \cos \pi/2 = 0$

$L(x) = 0 + (-1)(x - \pi/2)$

$= -x + \pi/2$

$L(1.75) = -1.75 + \pi/2$

$f(1.75) = -0.1782460556$

Error =

$\frac{|-0.1782460556 - (-1.75 + \pi/2)|}{-0.1782460556}$

$= -5.37 \times 10^{-3}$

Approximating Binomial Powers:

$$(1+x)^k \approx 1+kx$$

Use this formula to find polynomials that will approximate the following functions for values of x close to 0:

a. $\sqrt[3]{1-x}$

$\approx 1 + \frac{1}{3}(-x)$

$\approx 1 - \frac{1}{3}x$

b. $\frac{1}{1-x} = (1-x)^{-1}$

$\approx 1 + (-1)(-x)$

$\approx 1 + x$

c. $\sqrt{1+5x^4}$

$\approx 1 + (\frac{1}{2})(5x^4)$

$\approx 1 + \frac{5}{2}x^4$

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Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

~~(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your~~ answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

(a) $P'(9) \approx -0.646$ No the amount of Pollutants is decreasing at $t=9$ since $P'(t)$ is neg at that time

(b) $P'(t) = 0$
$t \approx 30.173724$

The number of gallons of pollutant will be a minimum at $t \approx 30.174$ b/c $P'(t)$ changes from neg to pos there and it is the only critical point

(c) $L(x) = f(a) + f'(a)(x-a)$
$(0, 50)$ $P'(0) = -2$
$L(x) = 50 - 2(x-0)$
$= 50 - 2x$

$40 \geq 50 - 2t$
$-10 \geq -2t$
$5 \leq t$
$t \geq 5$

when $p(t) \leq 40$
the model predicts that the lake becomes safe at day 5