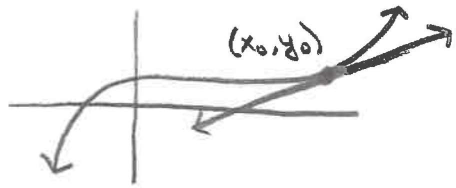
Use a tangent line

$$y - y_0 = m(x - x_0)$$

to approximate a function value



1. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

   $(2, 1)$ slope $= 4$

   $y - 1 = 4(x - 2)$

   $y = 4x - 7$

   $\boxed{f(1.9) \approx 0.6}$

   $\begin{array}{r} \overset{3}{1.9} \\ 4 \\ \hline 7.6 \end{array}$

   $y(1.9) = 4(1.9) - 7$
   $= 7.6 - 7$
   $= 0.6$

   extension: $(1.9, 0.6)$ under or over estimate?
   under b/c $f''(2) > 0$ so concave up

2. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

   $(1, 4)$ $f'(1) = 2(1) + 1 = 3$
   slope $= 3$

   $\boxed{f(1.2) \approx 4.6}$

   $y - 4 = 3(x - 1)$
   $y = 3x + 1$

   $y(1.2) = 3(1.2) + 1$
   $= 3.6 + 1$
   $= 4.6$

   extension: under or over estimate?
   under b/c $f''(x) = 2 > 0$ so f concave up @ $x = 1$

3. Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximation for $f(1.5)$ found by using the line tangent to the graph of f at $x = \frac{\pi}{2}$?

   $(\pi/2, \_\_)$ slope $= \_\_\_$

   $f(\pi/2) = 2\cos \pi/2 + 1$
   $= 1$

   $f'(x) = -2\sin x$
   $f'(\pi/2) = -2\sin \pi/2$
   $= -2$

   $\boxed{f(1.5) \approx -2 + \pi}$

   $y - 1 = -2(x - \pi/2)$

   $y = -2x + \pi + 1$

   $y(1.5) = -2(1.5) + \pi + 1$
   $= -2 + \pi$

4. Let f be the function defined by $f(x) = \sqrt[3]{x}$. What is the approximation for $f(10)$ found by using the line tangent to the graph of f at the point $(8, 2)$?

$(8, 2)$ $\quad f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

$f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$

$y - 2 = 1/12\,(x - 8)$

$y = 1/12\,x + 4/3$

$y(10) = \frac{1}{12}(10) + \frac{4}{3}$

$= 13/6$

$\boxed{f(10) \approx 13/6}$

5. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in (GL) and t is measured in days. The table below gives values of $W(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

$(30, 125)$ $\quad W'(30) = 1/2$

$y - 125 = 1/2\,(x - 30)$

$y = 1/2\,x + 110$

$y(32) = 1/2\,(32) + 110$

$= 16 + 110$

$= 126$

$\boxed{W(32) = 126}$

6. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$. Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

$h''(x) = \frac{x(2x) - (x^2 - 2)(1)}{x^2}$

$= \frac{2x^2 - x^2 + 2}{x^2}$

$= \frac{x^2 + 2}{x^2}$

$h''(4) = \frac{16 + 2}{16} > 0$ concave up

the line tangent to the graph of h @ x = 4 lies below b/c f''(4) > 0 so f is concave up