1. If $f(x) = 3x^4 - 2x^3 + 5x + 2$, find $f(4)$

a. Synthetic Substitution

4	3	-2	0	5	2
		12	40	160	660
	3	10	40	165	662

$f(4) = 662$

b. Direct Substitution

$f(4) = 3(4)^4 - 2(4)^3 + 5(4) + 2$

$= 3(256) - 2(64) + 20 + 2$

$= 768 - 128 + 20 + 2$

$= 662$

2. If $g(x) = 4x^5 + 2x^3 + x^2 - 1$, find $g(-1)$

a. Synthetic Substitution

-1	4	0	2	1	0	-1
		-4	4	-6	5	-5
	4	-4	6	-5	5	-6

$g(-1) = -6$

b. Direct Substitution

$g(-1) = 4(-1)^5 + 2(-1)^3 + (-1)^2 - 1$

$= -4 - 2 + 1 - 1$

$= -6$

3. Determine whether $x - 5$ is a factor of $x^3 - 7x^2 + 7x + 15$. Then find the remaining factors of the polynomial.

5	1	-7	7	15
		5	-10	-15
	1	-2	-3	0

$x - 5$ is a factor

$x^3 - 7x^2 + 7x + 15 = (x-5)(x^2 - 2x - 3)$

$= (x-5)(x-3)(x+1)$

4. Show that $x - 2$ is a factor of $x^3 - 7x^2 + 4x + 12$. Then find the remaining factors of the polynomial.

2	1	-7	4	12
		2	-10	-12
	1	-5	-6	0

$x^3 - 7x^2 + 4x + 12 = (x-2)(x^2 - 5x - 6)$

$= \boxed{(x-2)(x-6)(x+1)}$

* All zeros

$x = 2, 6, -1$

5. Given that $x+2$ is a factor of $x^3 - 3x + 2$, find the remaining factors of the polynomial.

$$\begin{array}{c|cccc} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$x^3 - 3x + 2 = (x+2)(x^2 - 2x + 1)$

$= (x+2)(x-1)(x-1)$

$= \boxed{(x+2)(x-1)^2}$

zeros: $x = -2, 1$ ← double zero

6. Give that $x-1$ is a factor of $x^4 + 2x^3 + 2x^2 - 2x - 3$, find the remaining factors of the polynomial.

and $x=-1$ is a zero

$$\begin{array}{c|ccccc} 1 & 1 & 2 & 2 & -2 & -3 \\ & & 1 & 3 & 5 & 3 \\ \hline & 1 & 3 & 5 & 3 & 0 \end{array}$$

$x^4 + 2x^3 + 2x^2 - 2x - 3 = (x-1)(x^3 + 3x^2 + 5x + 3)$

$$\begin{array}{c|cccc} -1 & 1 & 3 & 5 & 3 \\ & & -1 & -2 & -3 \\ \hline & 1 & 2 & 3 & 0 \end{array}$$

$= \boxed{(x-1)(x+1)(x^2+2x+3)}$

↑ prime

Review

Find each of the following:

a. Maxima

(-2, 3) & (3, 3)

b. Minima

(0, -2) & (3.5, -0.5)

c. Zeros

(-3.75, 0) (-1, 0) (1.5, 0) (3.25, 0)

d. Smallest possible degree of the function (3.75, 0)

x^5

e. Sign of the leading coefficient

pos

f. Domain

$(-\infty, \infty)$

g. Range

$(-\infty, \infty)$