2018 - 19

5.6 Related Rates

BC Calculus

1. Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4.5 = 4\pi (2)^2 \frac{dr}{dt}$

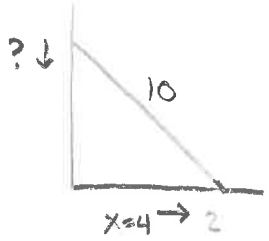
$4.5 = 16\pi \, dr/dt$

$\frac{dV}{dt} = 4.5$

$r = 2$

$0.0895 \text{ ft/min} = dr/dt$

2. A ladder 10 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when its base is 4 ft. from the wall?



$4^2 + y^2 = 10^2$

$16 + y^2 = 100$

$y^2 = 84$

$y = \sqrt{84}$

$x^2 + y^2 = 10^2$

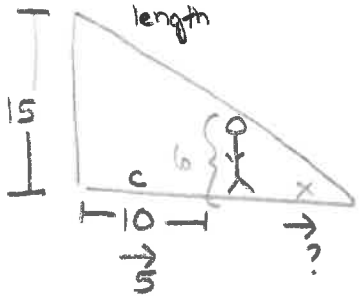
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(4)(2) + 2(\sqrt{84})\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{16}{2\sqrt{84}}$

$= -0.873 \text{ ft/sec}$

3. A man 6 feet tall walks at the rate of 5 feet per second away from a light that is 15 feet above the ground when he is 10 feet from the base of the light, at what rate is the tip of his shadow moving?



$\frac{15}{c+x} = \frac{6}{x}$

$15x = 6c + 6x$

$9x = 6c$

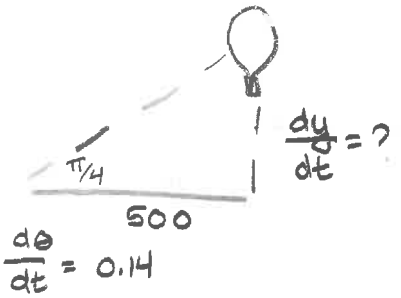
$9\frac{dx}{dt} = 6\frac{dc}{dt}$

$\frac{dx}{dt} = \frac{6(5)}{9}$

$= 10/3 \text{ ft/sec}$

* 10 is not set ⇒ changes

4. A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the liftoff point. At the moment the range finder's elevation angle is , $\frac{\pi}{4}$ the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?



$$\tan\theta = \frac{y}{500}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{500}\frac{dy}{dt}$$

$$500 \sec^2 \pi/4 \,(0.14) = \frac{dy}{dt}$$

$$140 \text{ ft/min} = $$

5. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with the radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

$$0.6^2 + 0.8^2 = h^2$$

$$1 = h$$

$$\frac{dp}{dt} = -60$$

$$\frac{dc}{dt} = ?$$

$$p^2 + c^2 = h^2$$

$$2p\frac{dp}{dt} + 2c\frac{dc}{dt} = 2h\frac{dh}{dt}$$

$$2(0.6)(60) + 2(0.8)\frac{dc}{dt} = 2(1)(20)$$

$$-72 + 1.6\frac{dc}{dt} = 40$$

$$1.6\frac{dc}{dt} = 112$$

$$= 70 \text{ mph}$$